VIS 2003

Subdivision Methods for Volume Meshes

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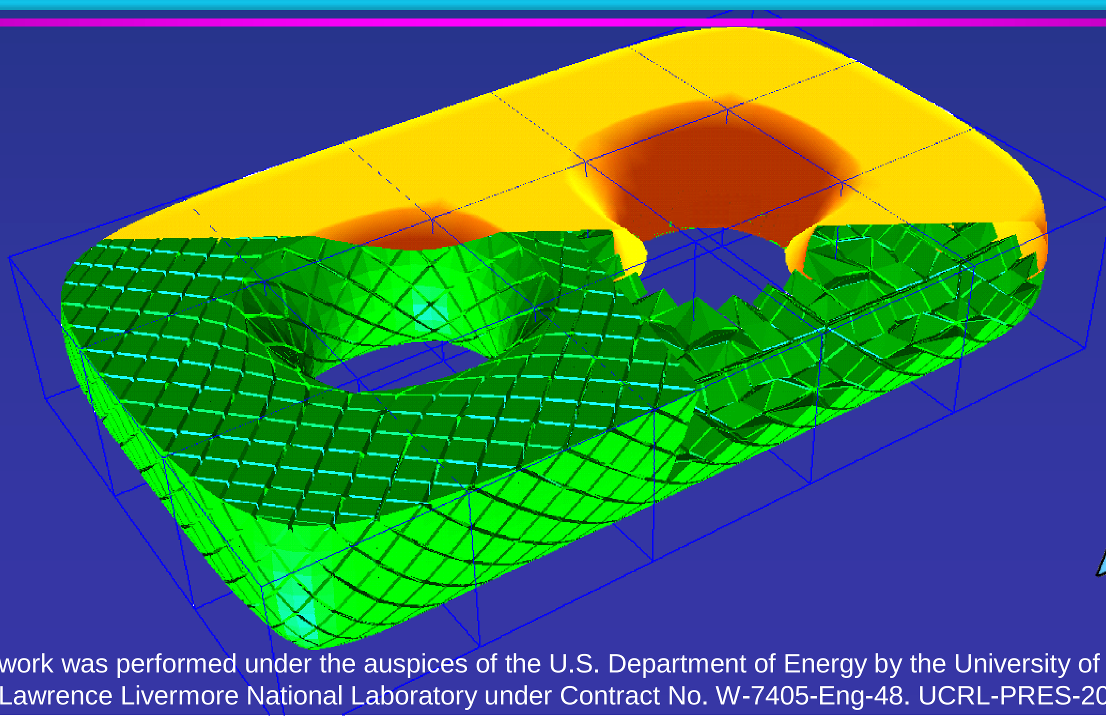
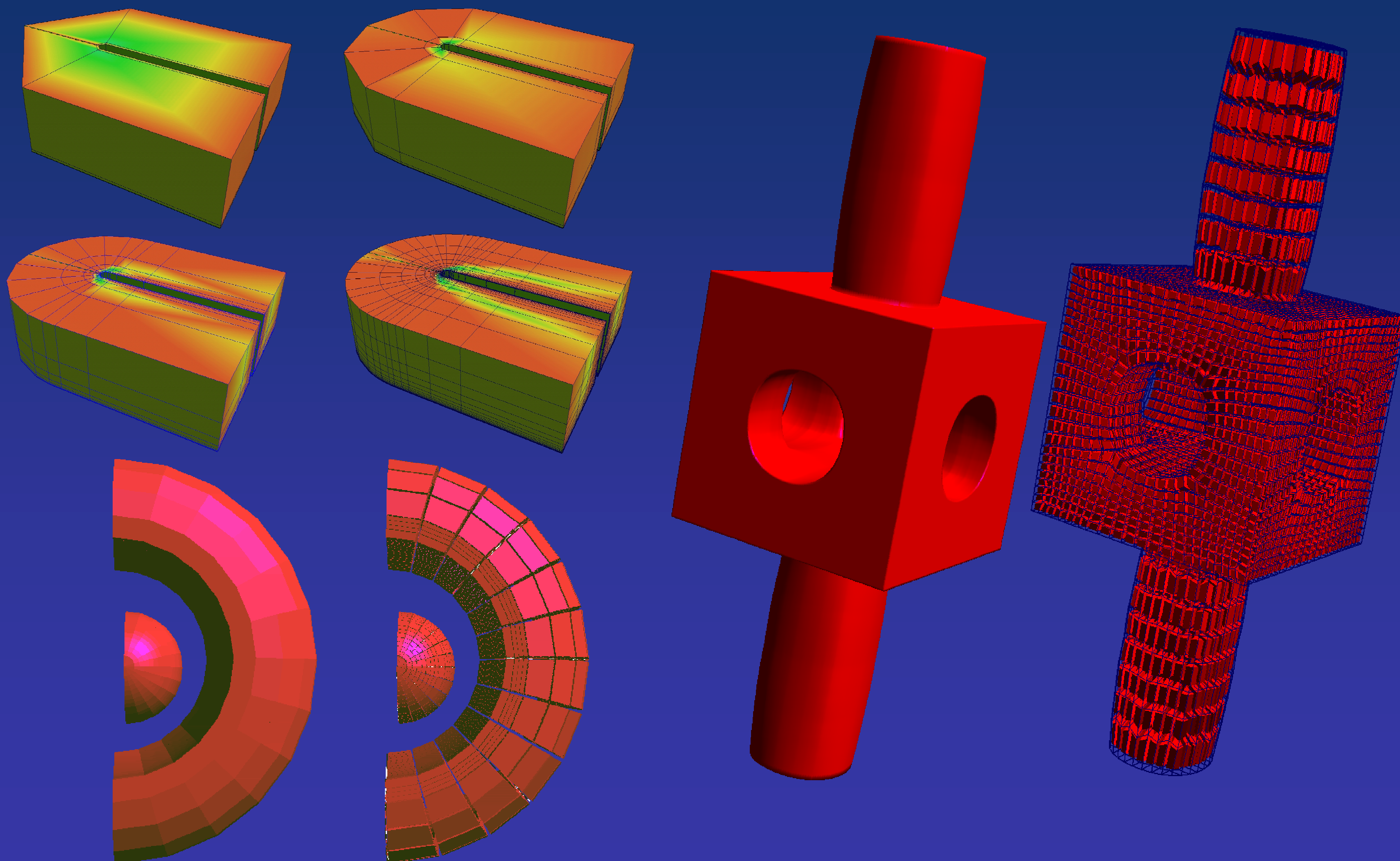


This work was performed under the auspices of the U.S. Department of Energy by the University of California, Lawrence Livermore National Laboratory under Contract No. W-7405-Eng-48. UCRL-PRES-200744

We consider efficient schemes for representing multi-scale volumes.



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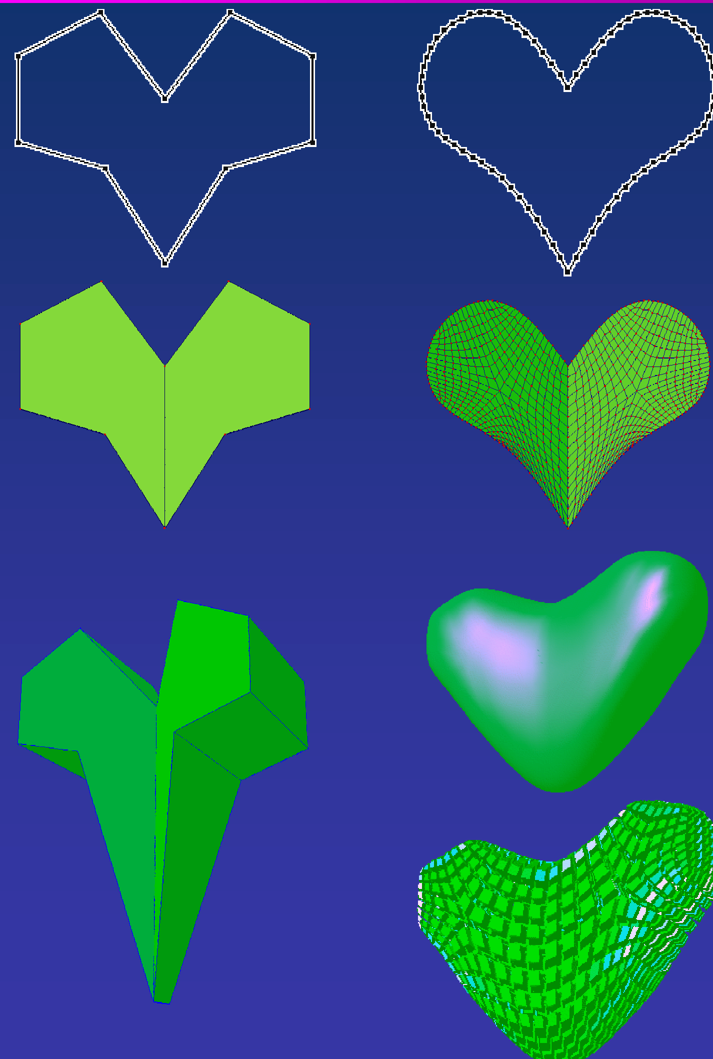
Subdivision from a combinatorial and dimensional point of view.

- 1D B-spline subdivision
- Multi-linear subdivision: B-spline refinement for hexahedral domains
- 4-8 subdivision
- 3D Slow Growing Subdivision: easy management of adaptive refinement and sharp features



Subdivision Methods Have Been A Success For Surface Meshes

- **Fast high quality rendering**
  - "Subdivision Surfaces in Character animation", T.DeRose, M.Kass & T.Truong, '98.
- **Multi-resolution representation**
  - "Normal Meshes", I. Guskov, K.Vidimce, W.Sweldens & P.Schröder, '00.
  - "Displaced Subdivision Surfaces", A. Lee, H. Moreton & H. Hoppe, '00.
- **Progressive Compression**
  - "Progressive Geometry Compression", A. Khodakovsky, P. Schröder & W.Sweldens, '00.
- **Controlled smoothness of limit surface**
  - "Smoothness of subdivision on irregular meshes", D. Zorin, '00.
- **Vertex proliferation problem already arises**
  - "$\sqrt{3}$—Subdivision", L. Kobbelt, '00.

Volumetric Meshes Are Fundamentally More Challenging

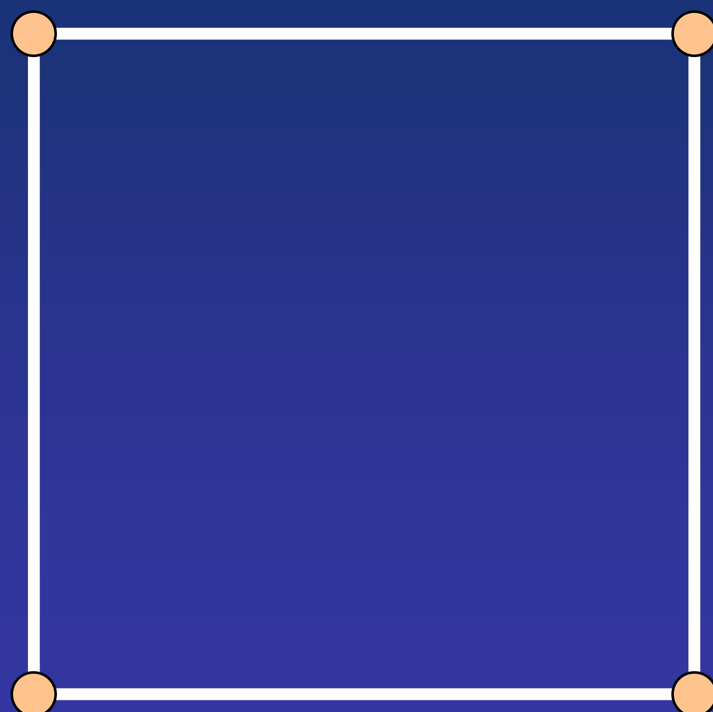
- **Combinatorial Problem:**
  tensor product generalizations are not satisfactory
  - Excessive proliferation of vertices per refinement
  - Adaptive refinement requires special cells
  - Restricted types of base meshes

  📖 Pascucci, '02, "Slow Growing Subdivision in Any Dimension".

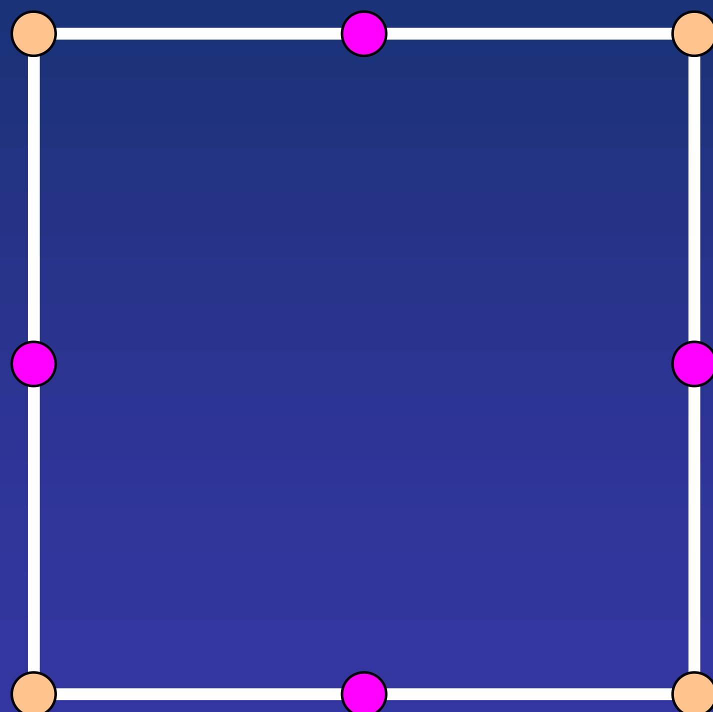
- **Numerical Problem:**
  smoothness analysis is hard in general
  - combinatorial explosion of the complexity at the extraordinary points

📖 C.Bajaj, J. Warren, & G. Xu, '02, "A smooth subdiv. scheme for hexaedral meshes".
📖 R.MacCracken & K. Joy, '96, "Free-form deform. with lattices of arbitrary topology".
📖 M. Bertram, '02, "Biorthogonal Wavelets for Subdivision Volumes".
📖 Y.-C. Chang et al. "A New Subdivision Scheme Based on Box Splines".
📖 L.Linsen et al., '03, "Wavelet-based multi-resolution with n-th-root-of-2 subdivision".

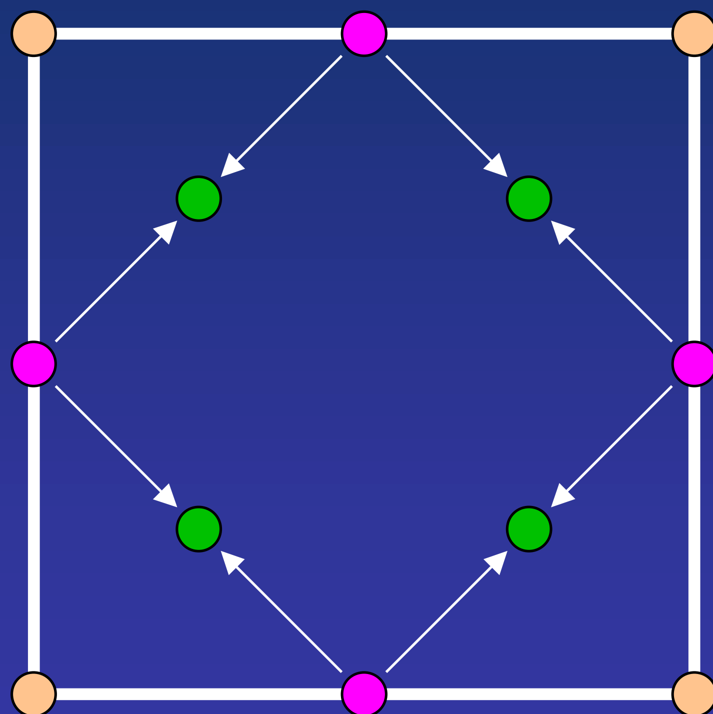
B-spline polygon subdivision



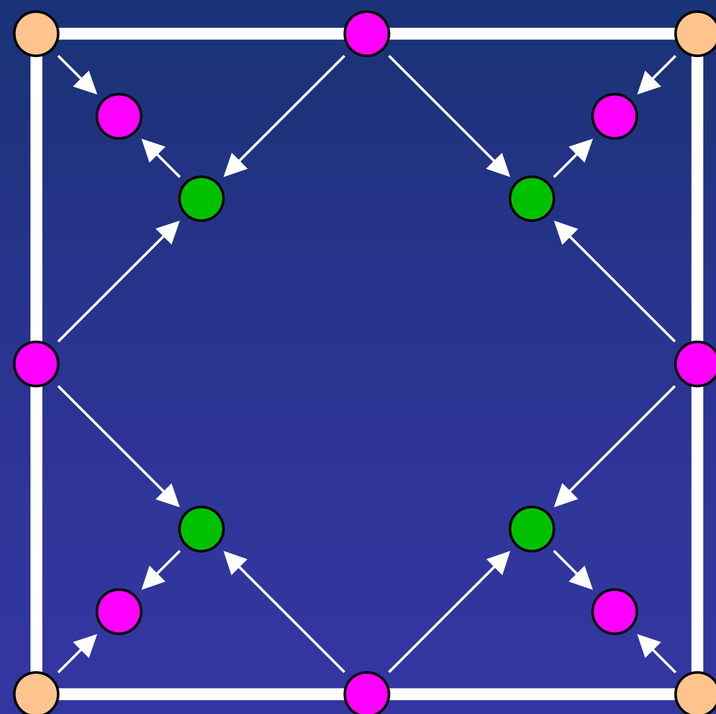
B-spline polygon subdivision



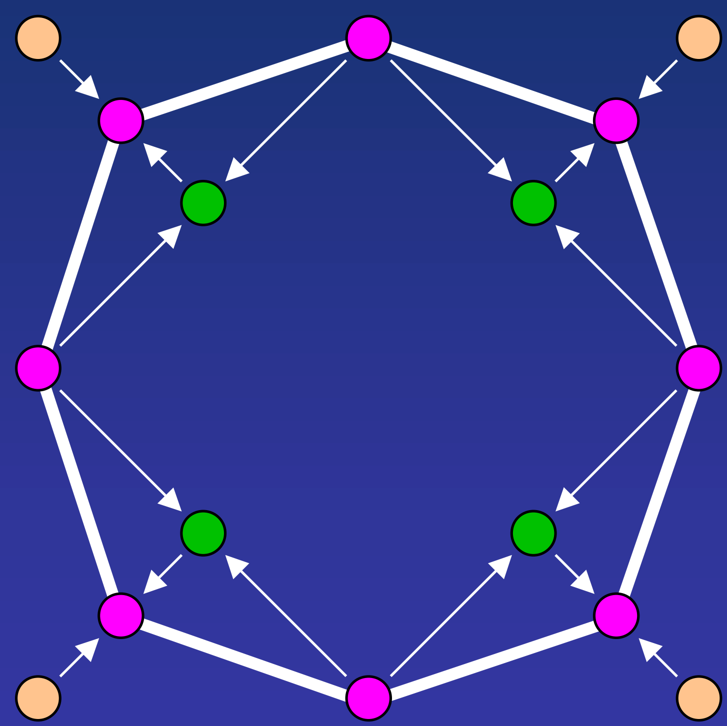
B-spline polygon subdivision



B-spline polygon subdivision



B-spline polygon subdivision



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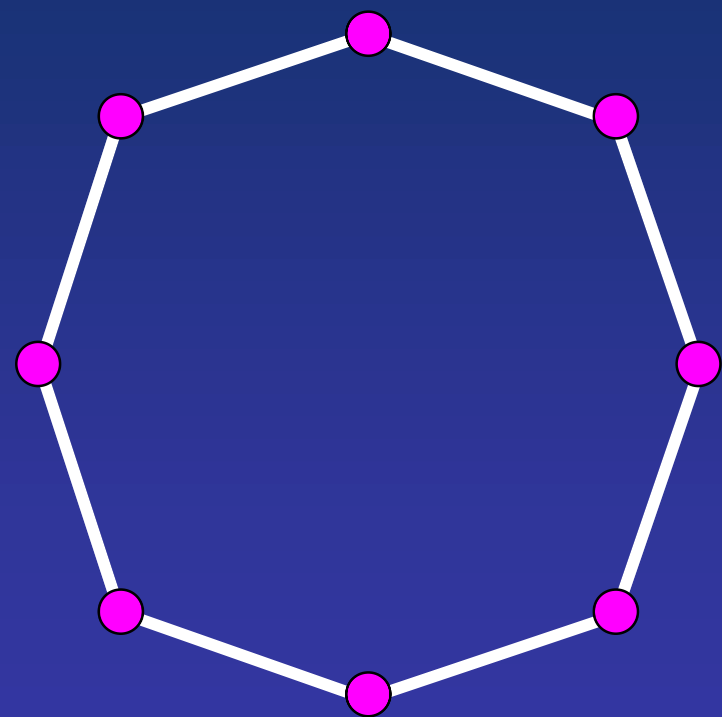
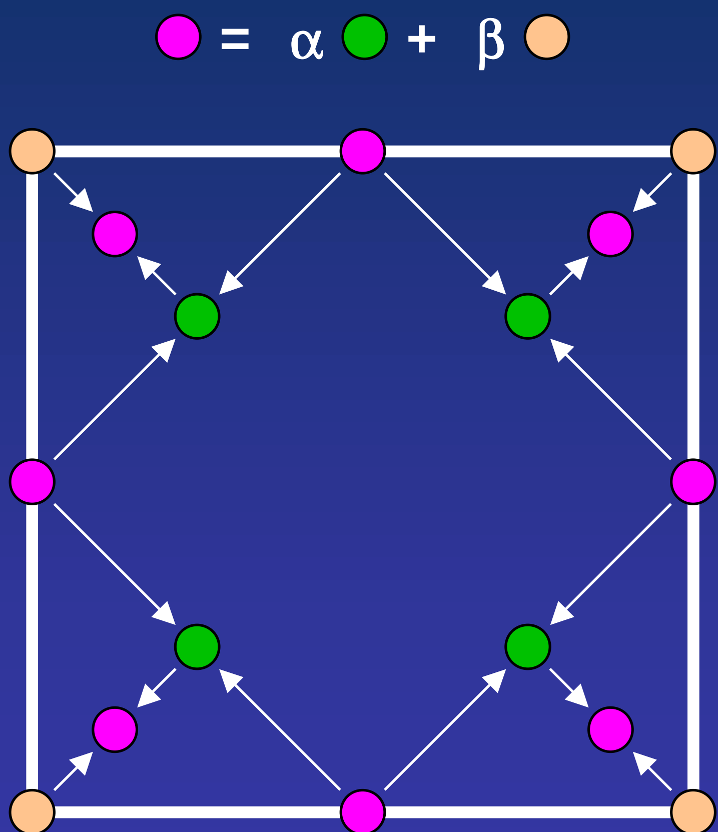
B-spline polygon subdivision

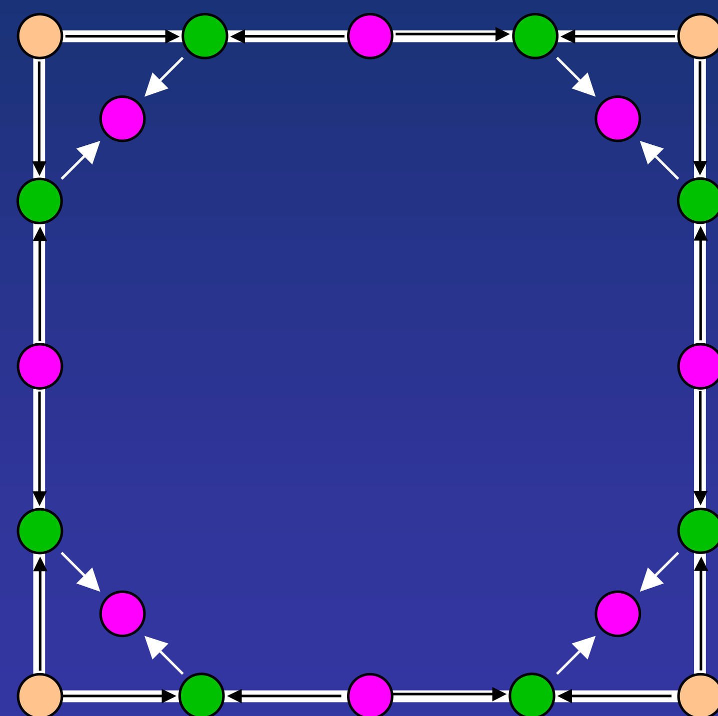


Possible variations on the 1D B-spline subdivision



Allow alternative shapes

average of edge centroids



Great for dimensional generalization!

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2D multi-linear refinement.

- **Combinatorics:** tensor product refinement of each cell.
- **Smoothing:** vertices displaced to the average of the centroids of the incident cells.

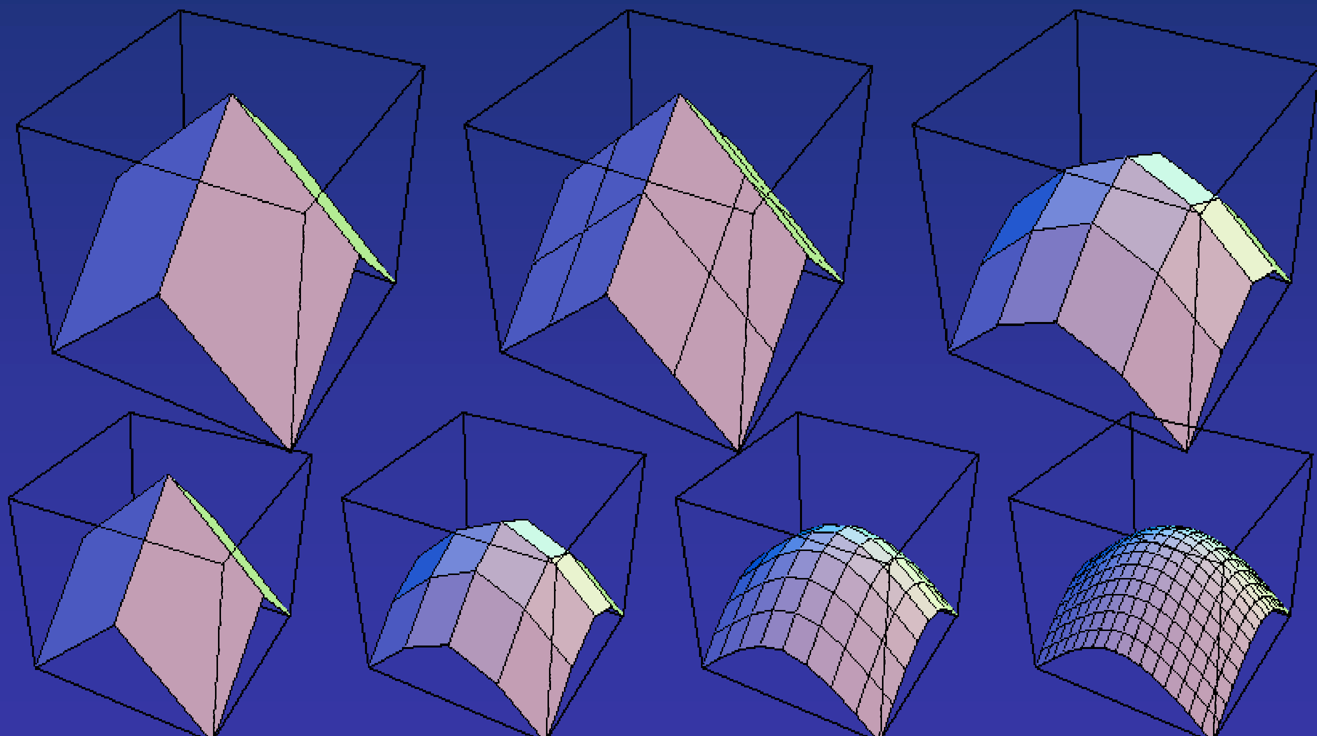


Figure courtesy of Joe Warren.

3D multi-linear refinement.

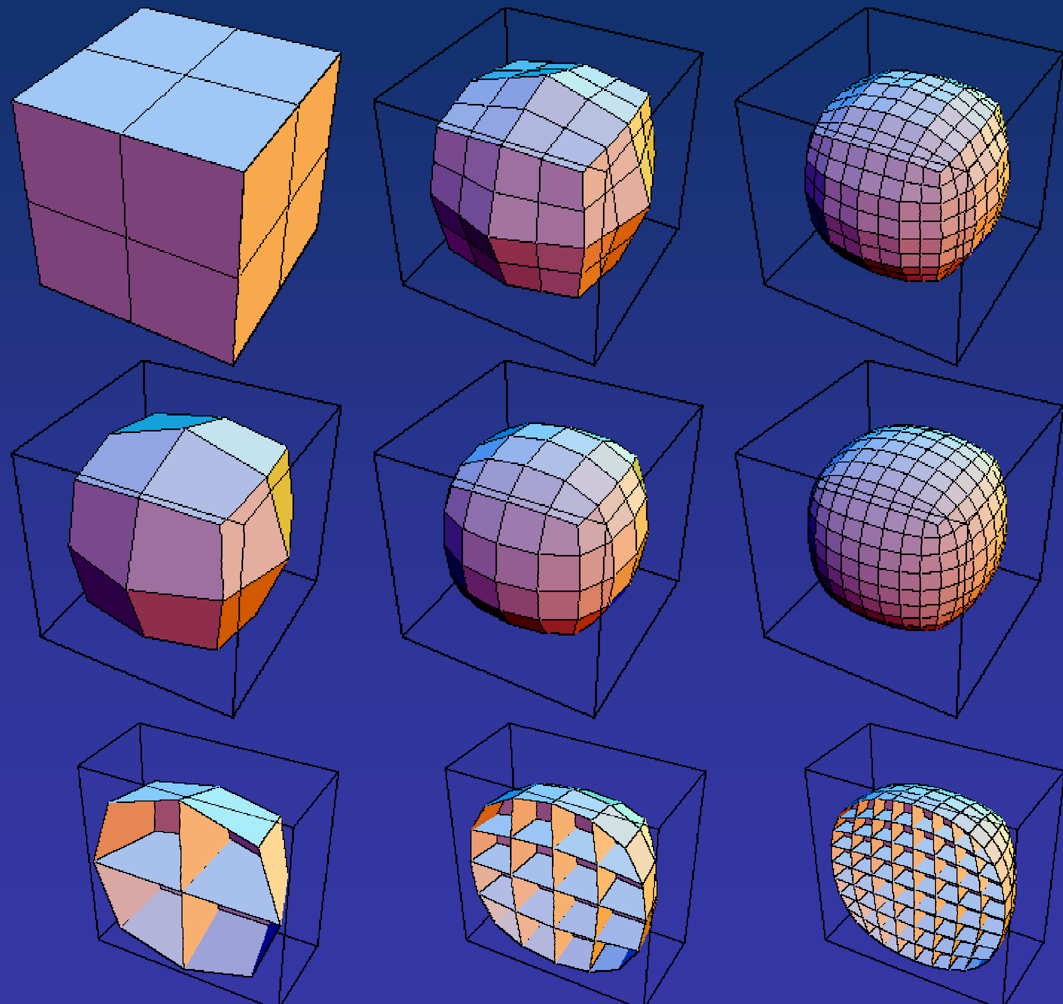


Figure courtesy of Joe Warren.

Best known smoothness properties in the 3D case.

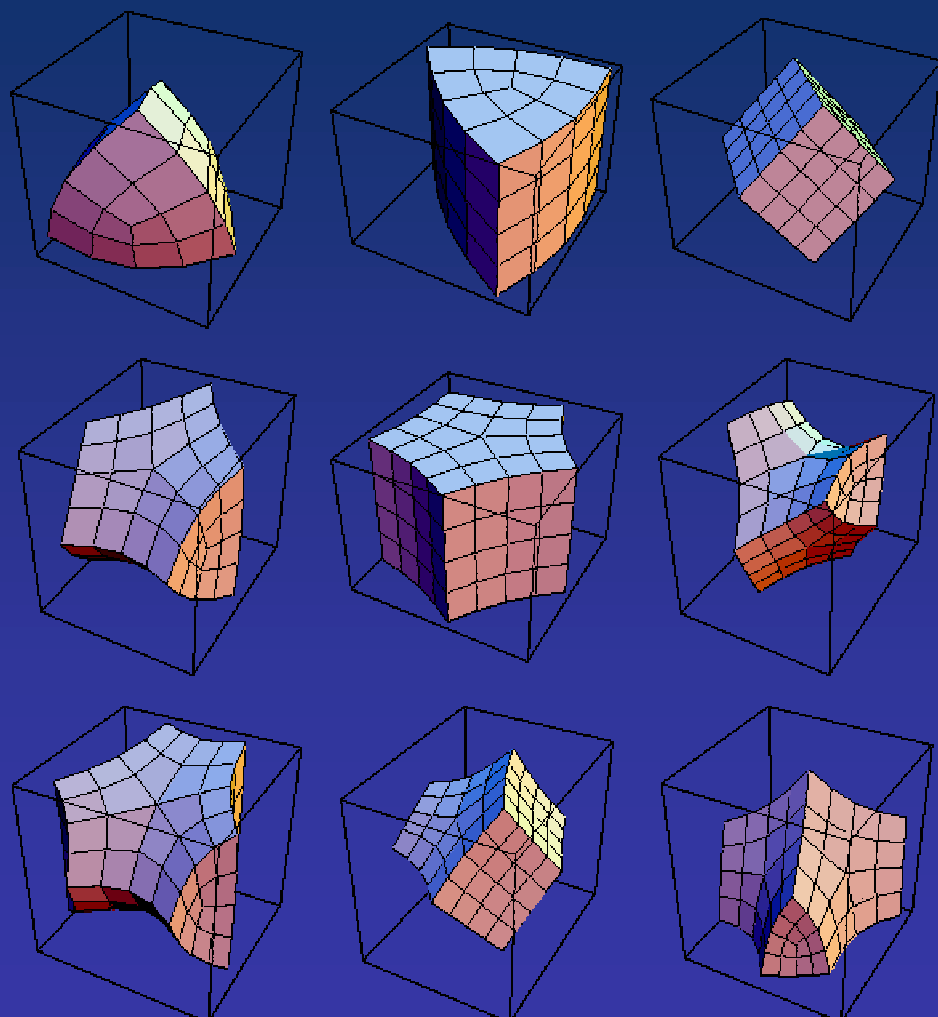
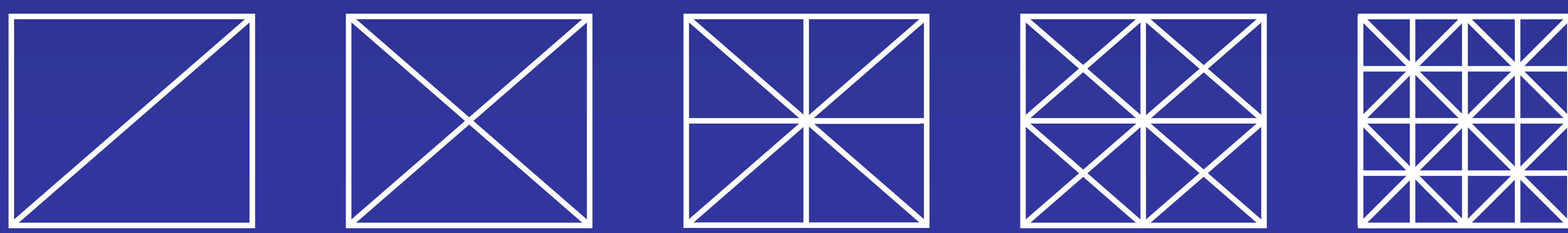


Figure courtesy of Joe Warren.

Longest Edge Bisection (Grid): 4-8 Mesh = $\sqrt{2}$ subdivision

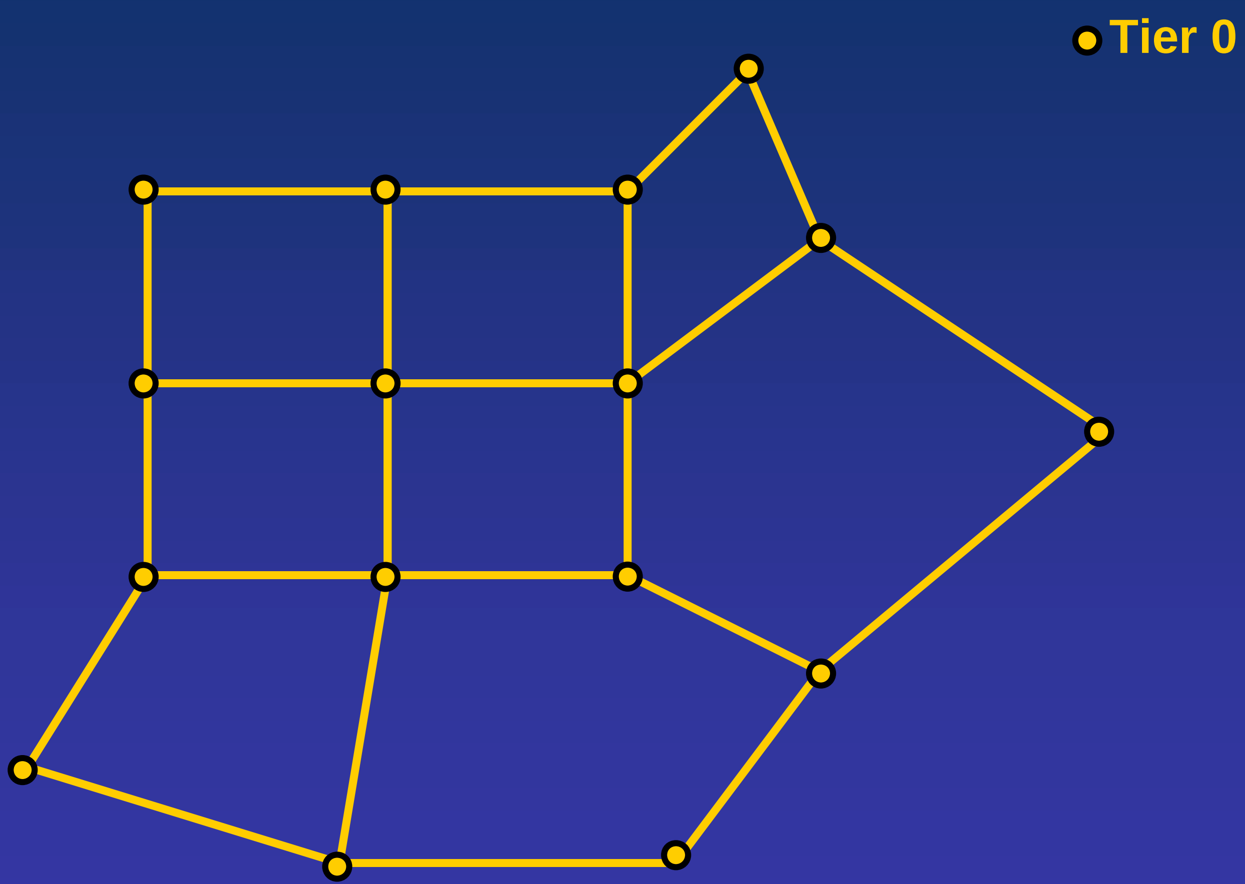
- "Algorithms for refining triangular grids suitable for adaptive and multigrid techniques", M.-C.Rivara, '84.
- "4-8 subdivision", L.Velho & D.Zorin, '01.
- "Smooth centroid bin-tree subdivision surfaces with local wavelets", M.Duchaineau, B.Gregorski, & K.Joy, '01.



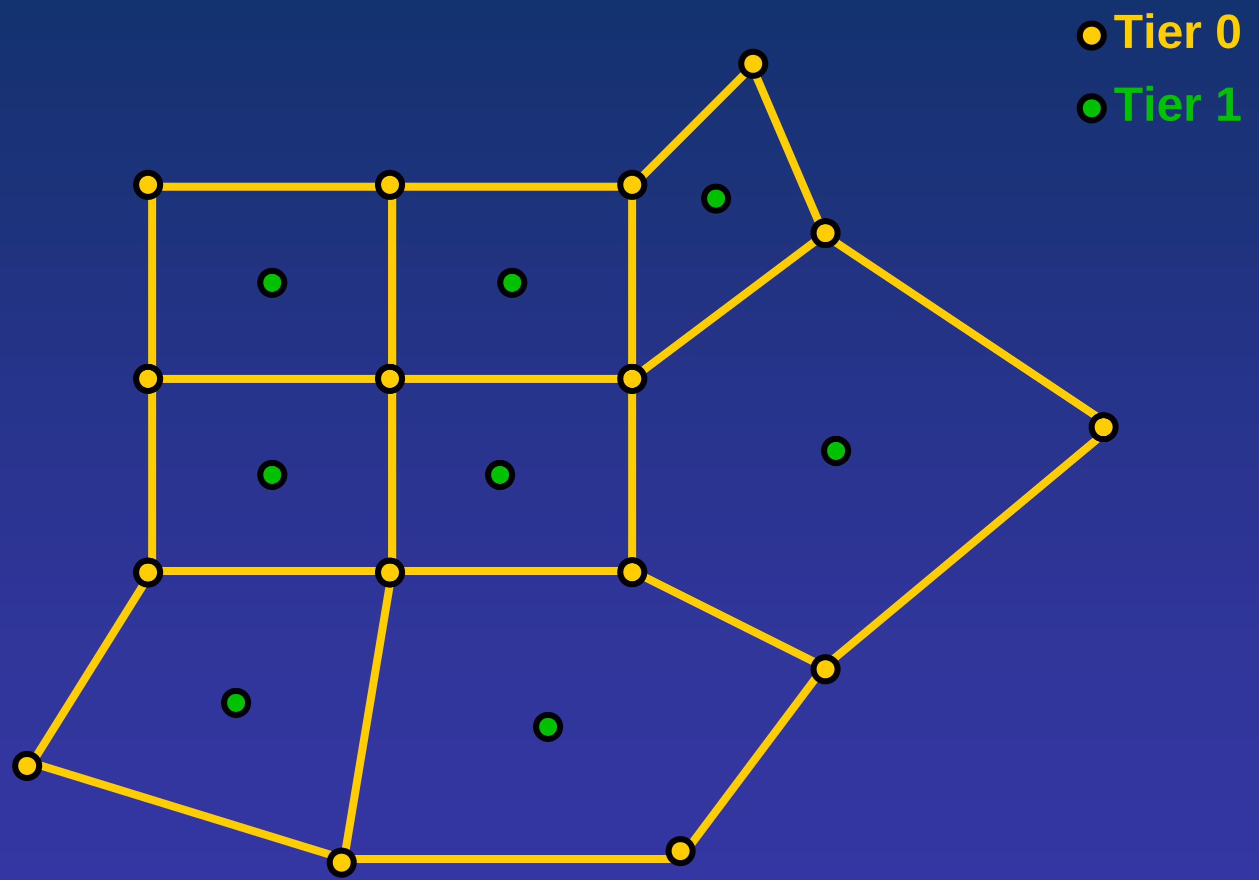
SGS Solves the Combinatorial Side of the Problem

- **Very general type of coarse mesh**
- **No special treatment of adaptive refinement**
- **"Sharp" features: independent refinement of lower dimensional sub-meshes**
- **Rate of refinement almost independent of dimension: only double the number of vertices at each refinement**
  - "A 3-d refinement algorithm suitable for adaptive and multi-grid techniques", M.C. Rivara & C.Levin, '92.

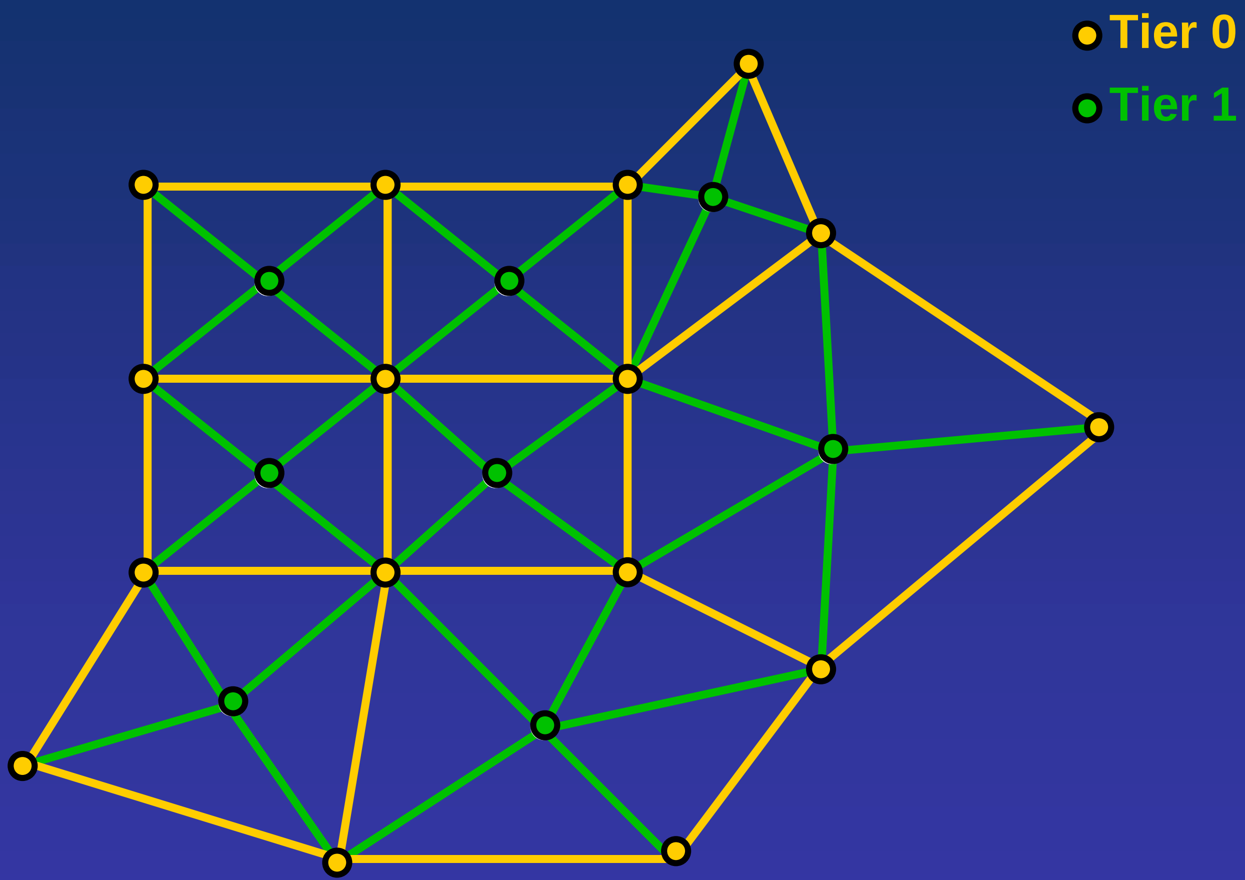
2D Slow Growing Subdivision: Base Mesh



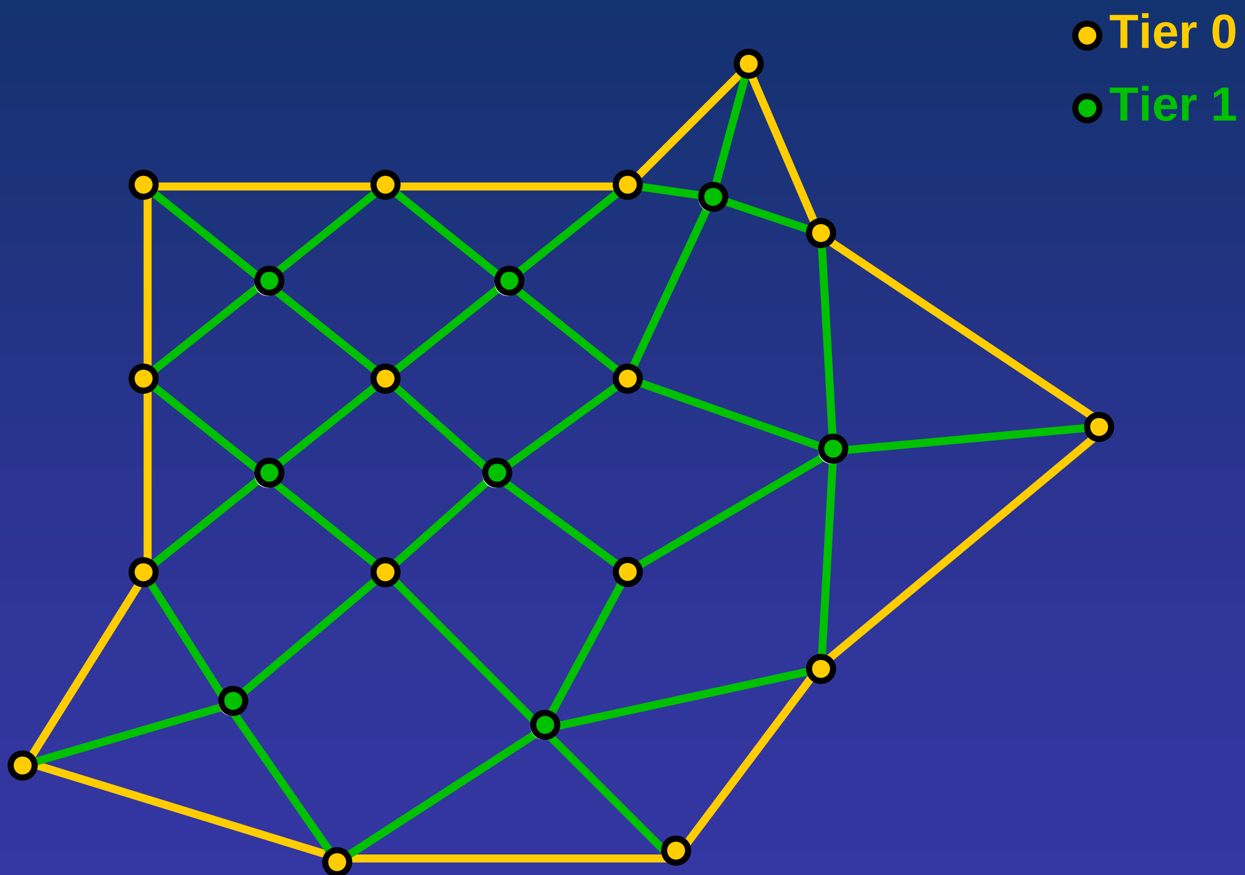
Compute the Centroid of Each 0-Face: 1-Vertices



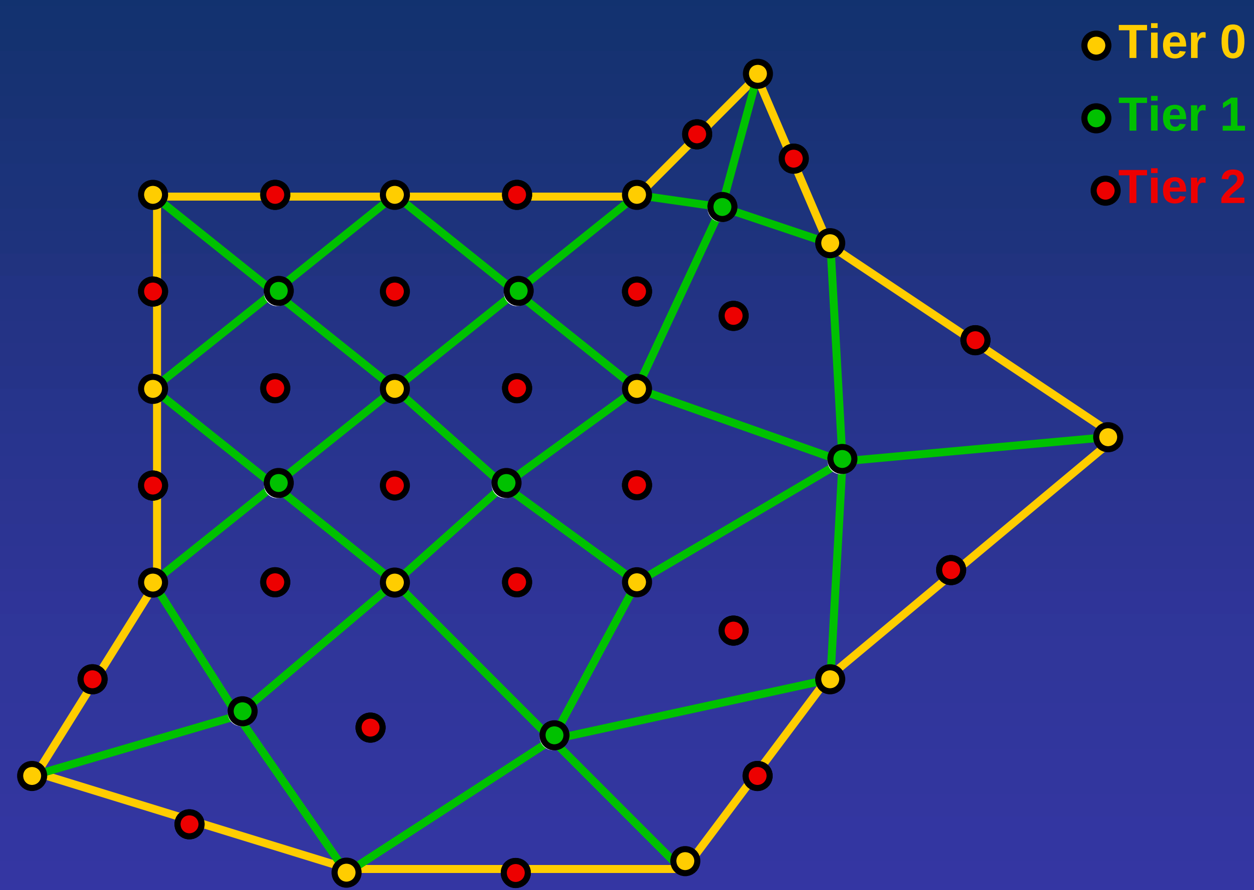
Connect each 1-Vertex To the Adjacent Vertices



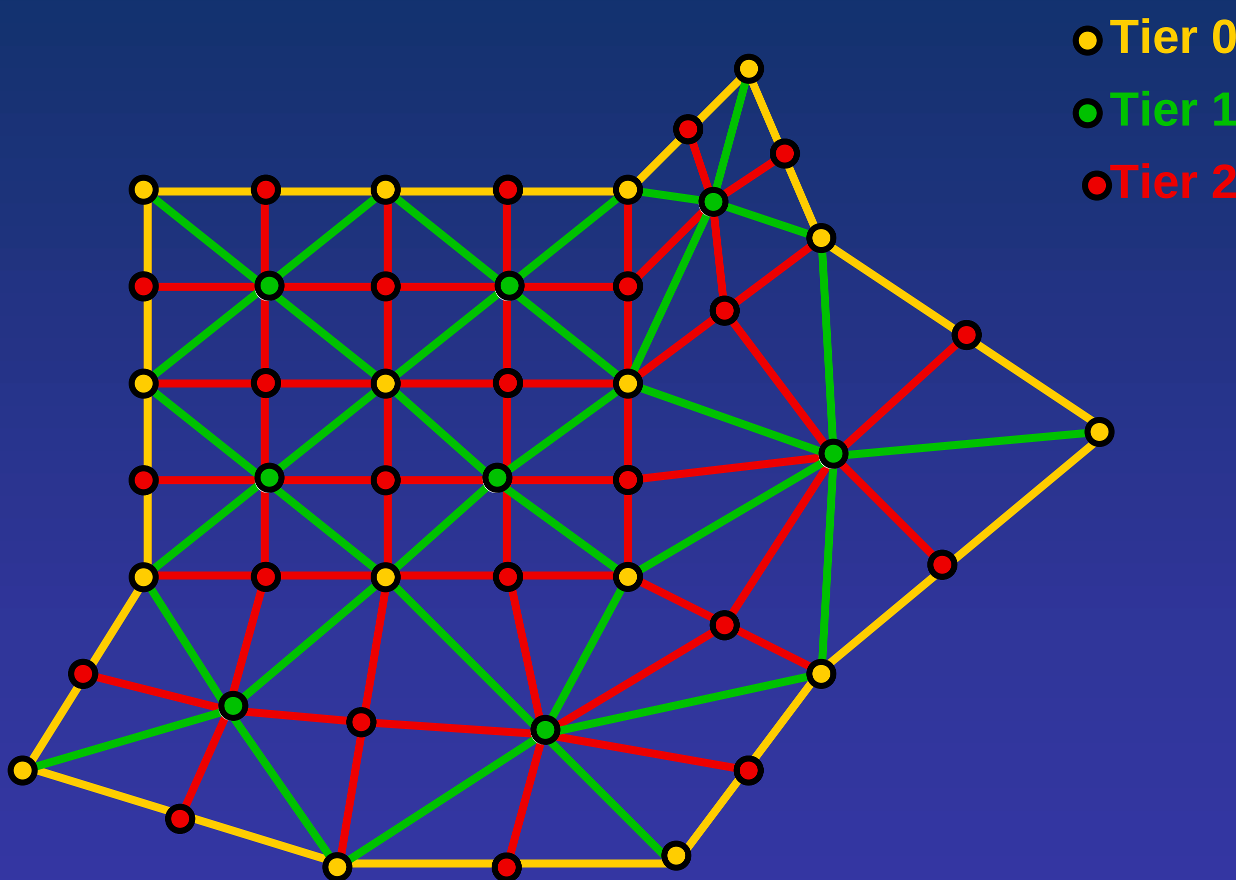
Merge Each Pair of Triangles That Share a 0-Edge



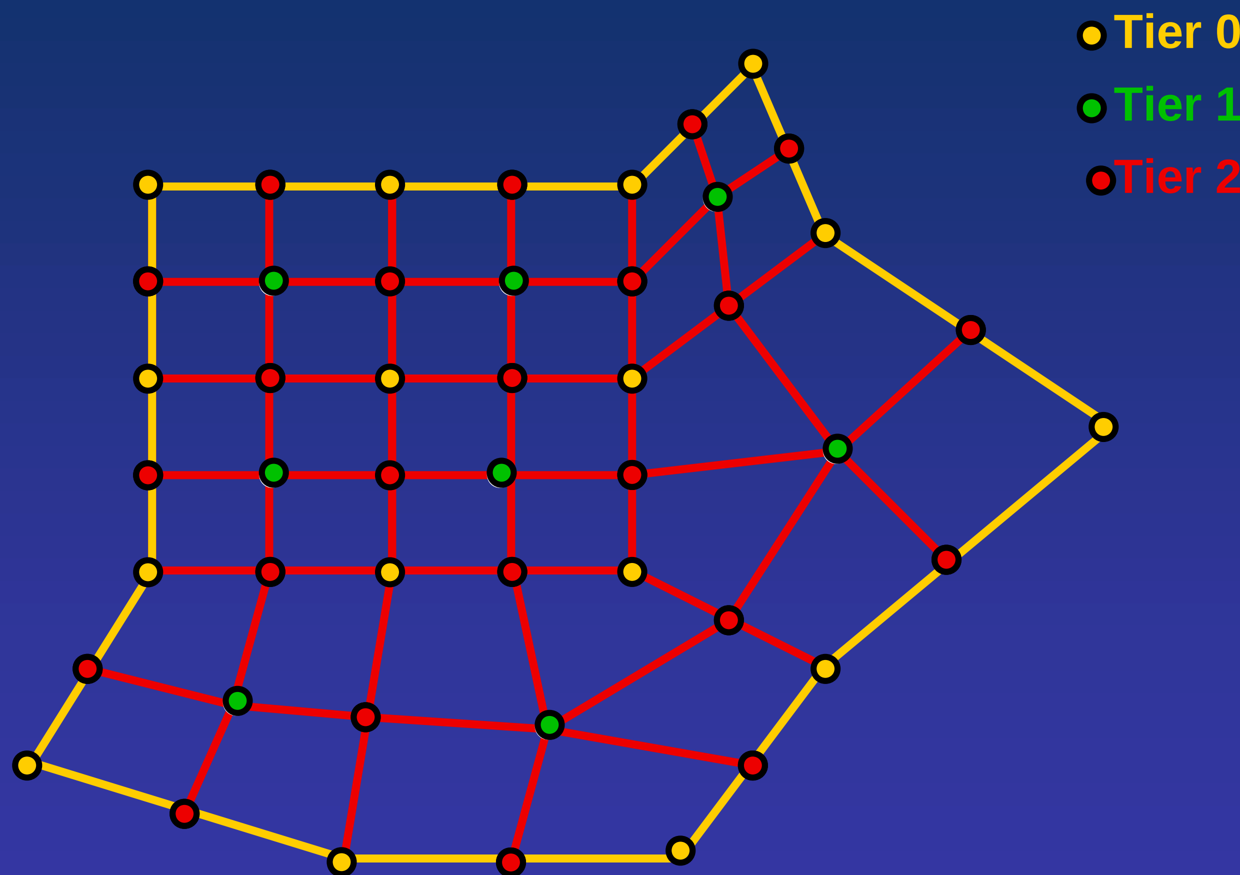
Compute the Centroid of Each 0-Edge/1-Face: 2-Vertices



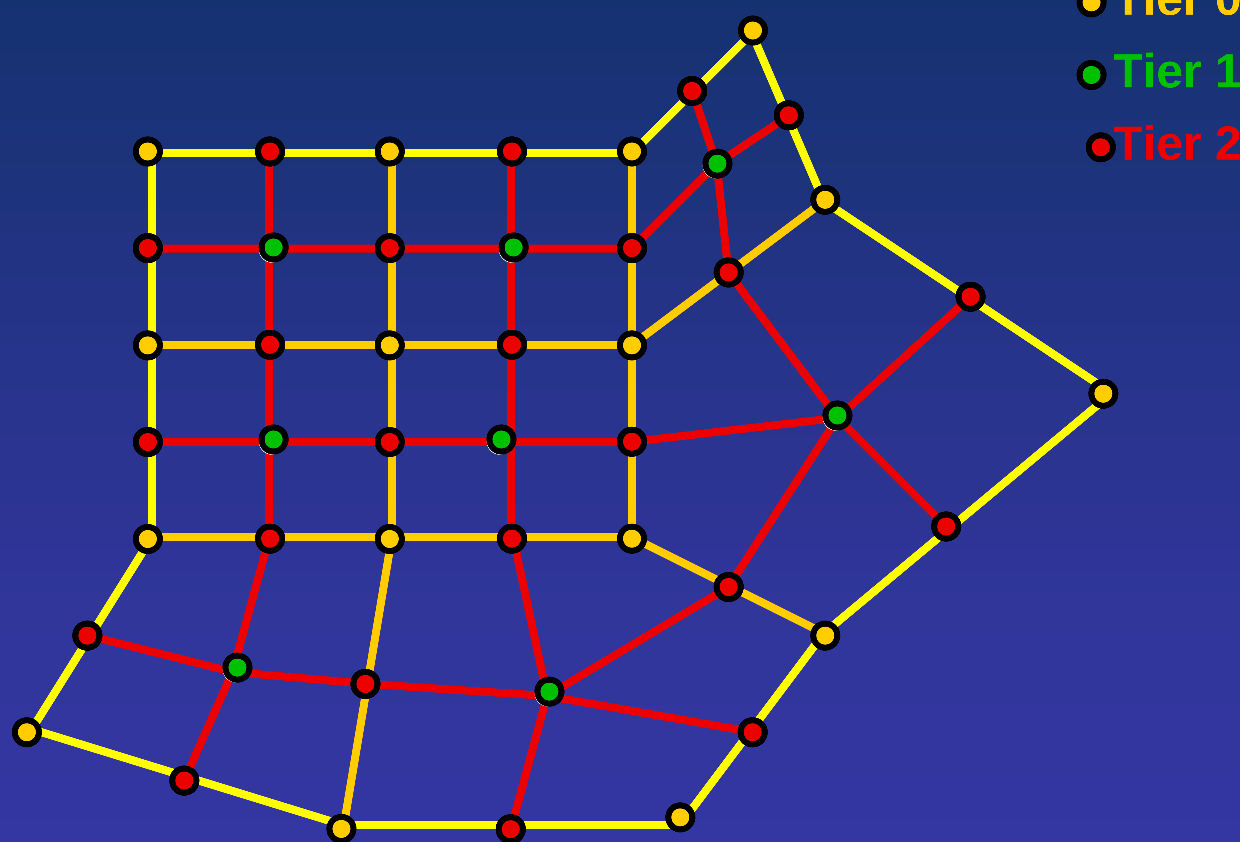
Connect each 2-Vertex to the Adjacent Vertices



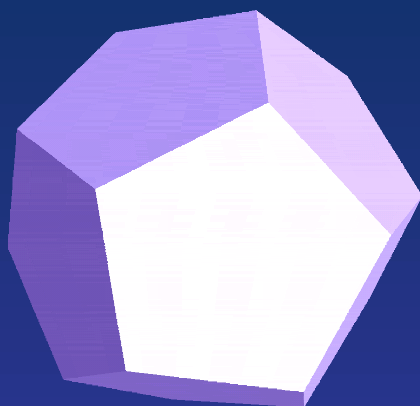
Merge Each Pair of Triangles in a 0-Face Sharing a 0-Vertex



2 Refinements Needed to Bisect All Edges ⇒ $\sqrt{2}$—subdivision

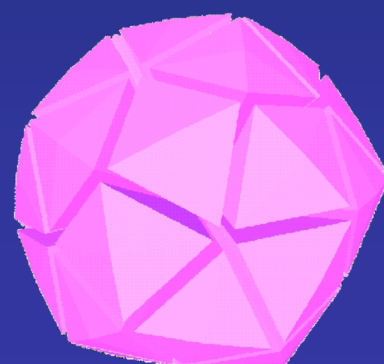


It is important to have a scheme that Applies to General Cells.



Tier 0 (base mesh)

Tier 1

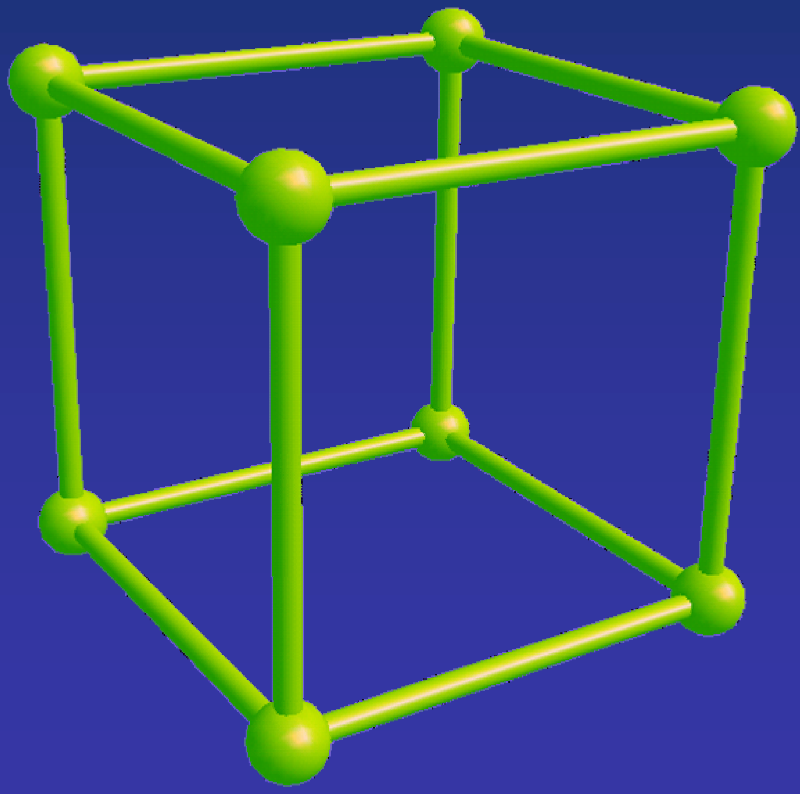


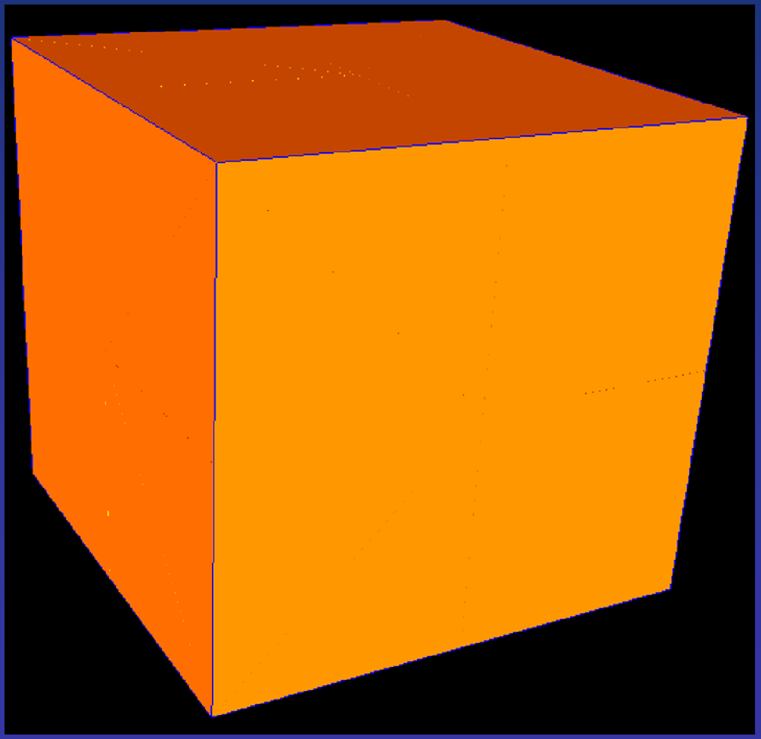
Tier 2

Tier 3

3D Slow Growing Subdivision for Exahedral Cells

- **Tier 0**





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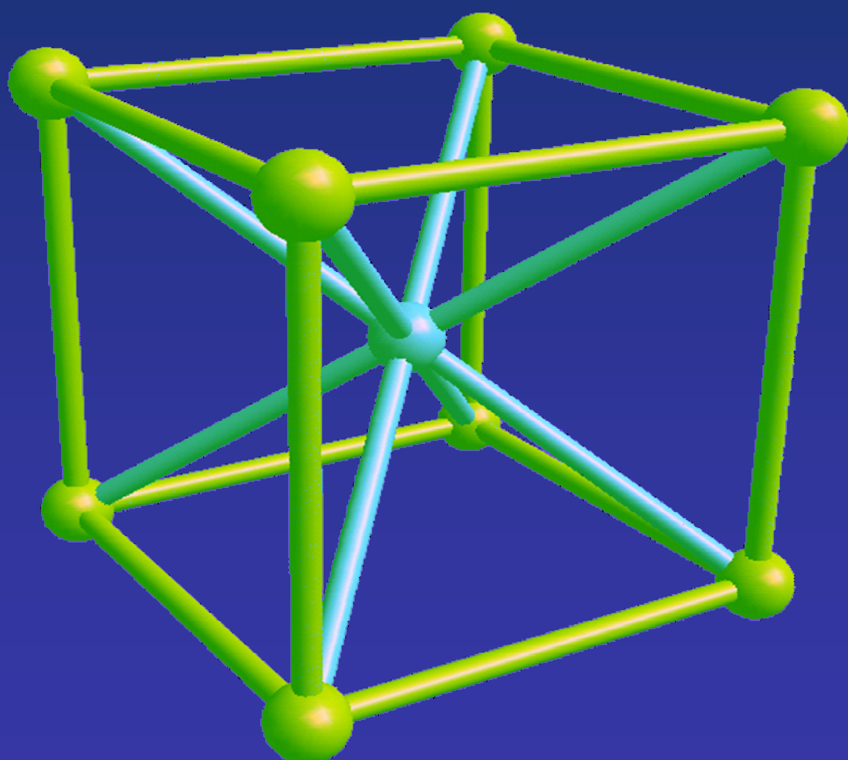
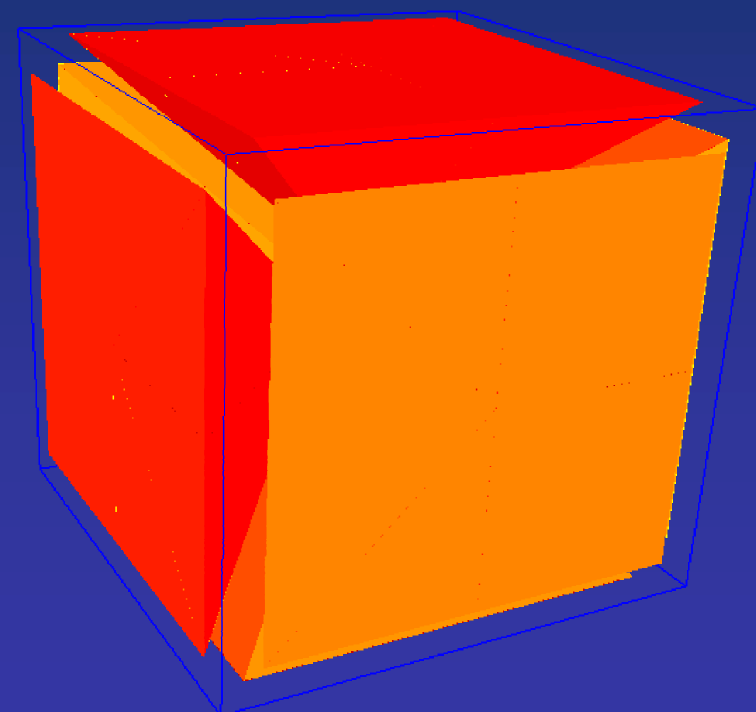
Compute the Centroid of Each Cell and Connect to Each Face

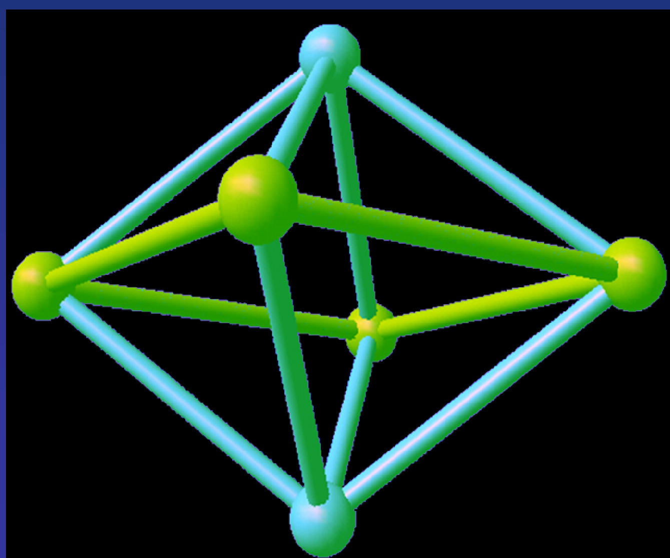
- Tier 0
- Tier 1

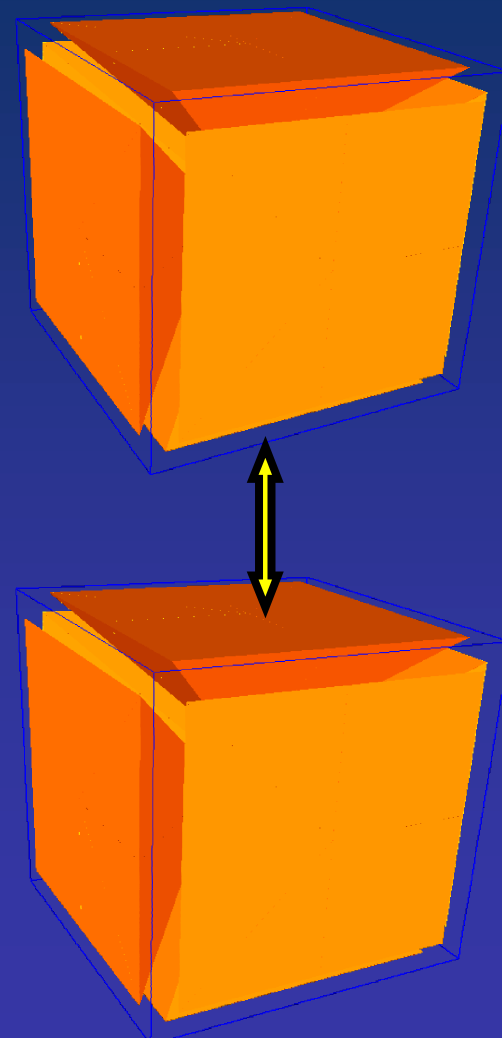




6 face pyramids

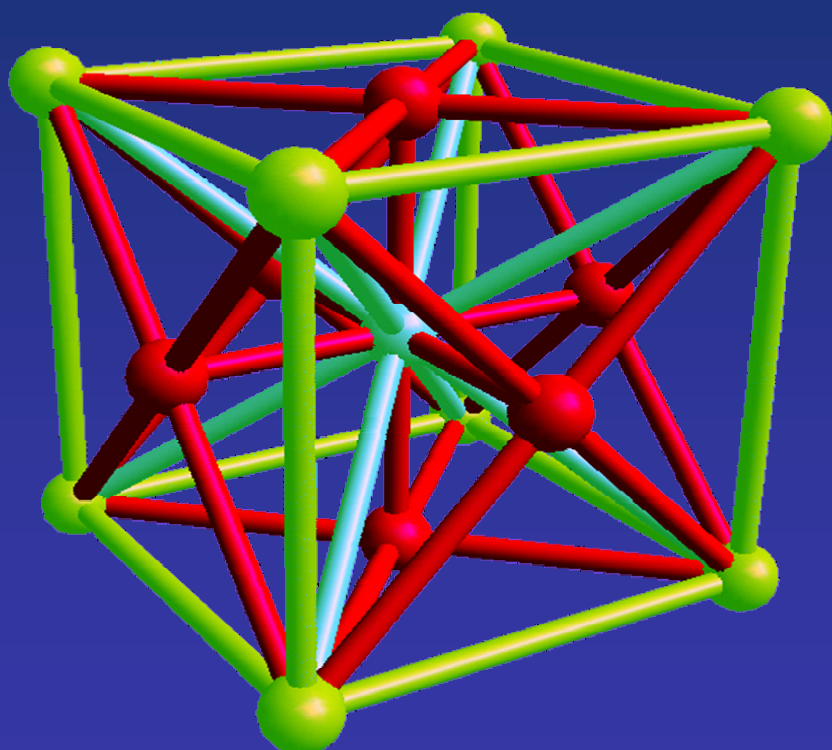
Merge Two Pyramids on the Common Face ….. If Possible





Compute New Centroids and Connect to Each Face

- Tier 0
- Tier 2
- Tier 1



The new pyramids are tetrahedra

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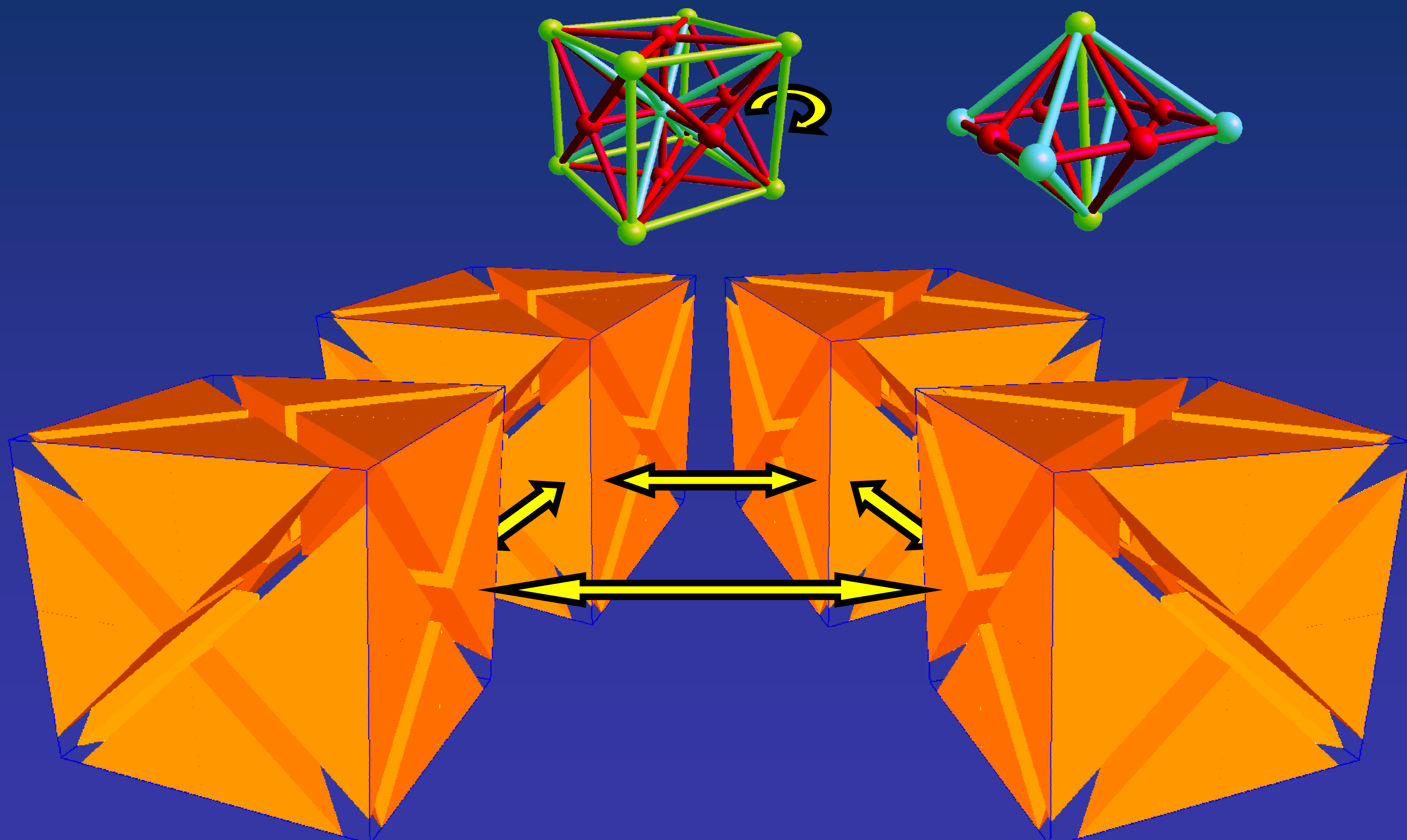
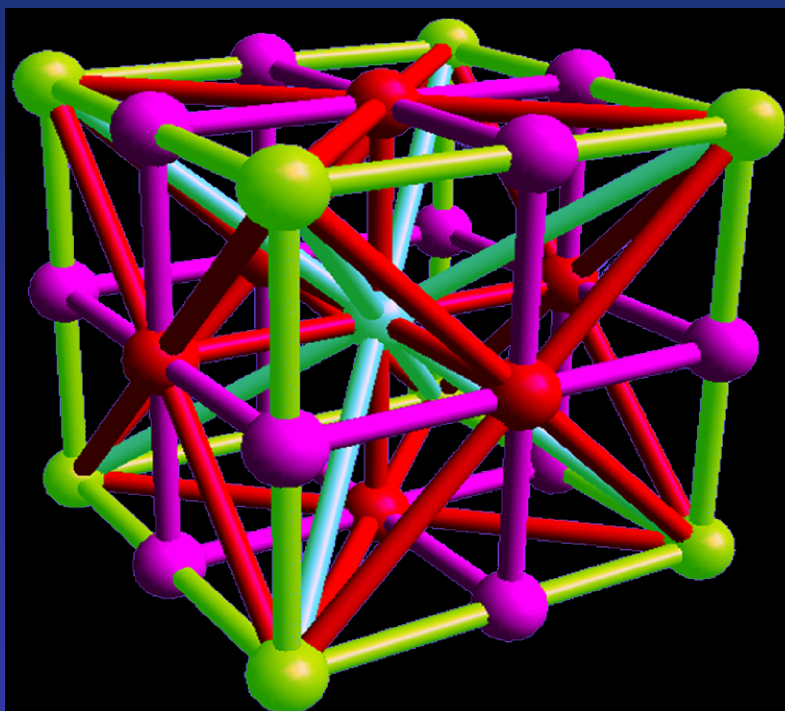
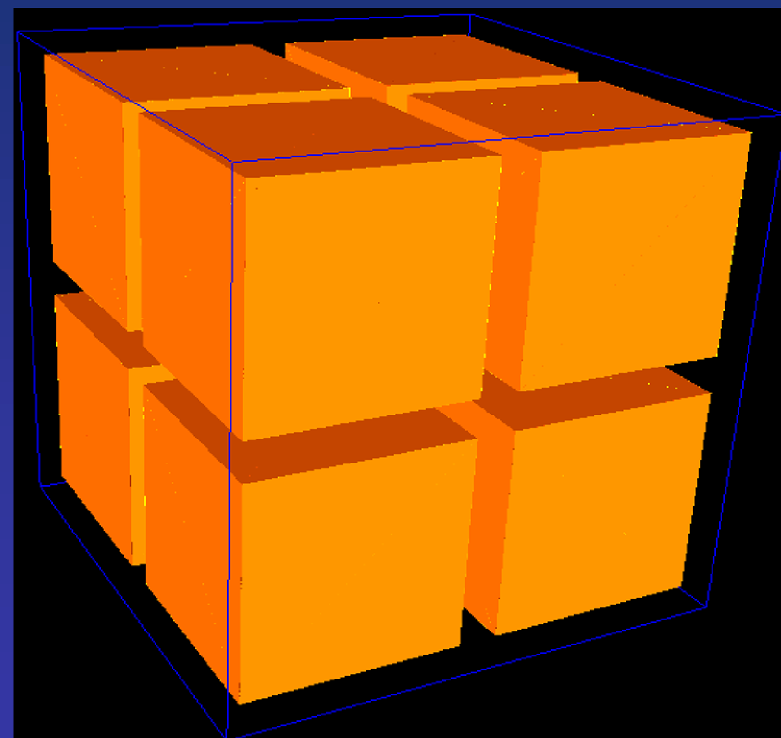
Merge the Tetrahedra on the Common Edge ….. If Possible



Add New Centroids, Connect to Each Face and Merge Tetrahedra

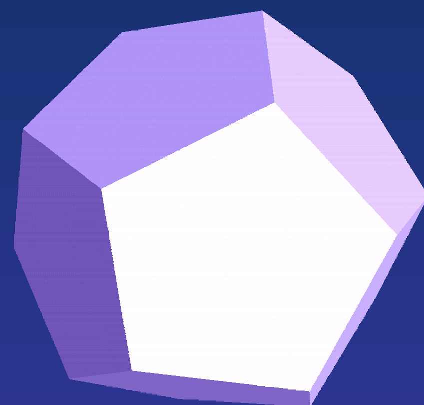
- Tier 0
- Tier 1
- Tier 2
- Tier 3



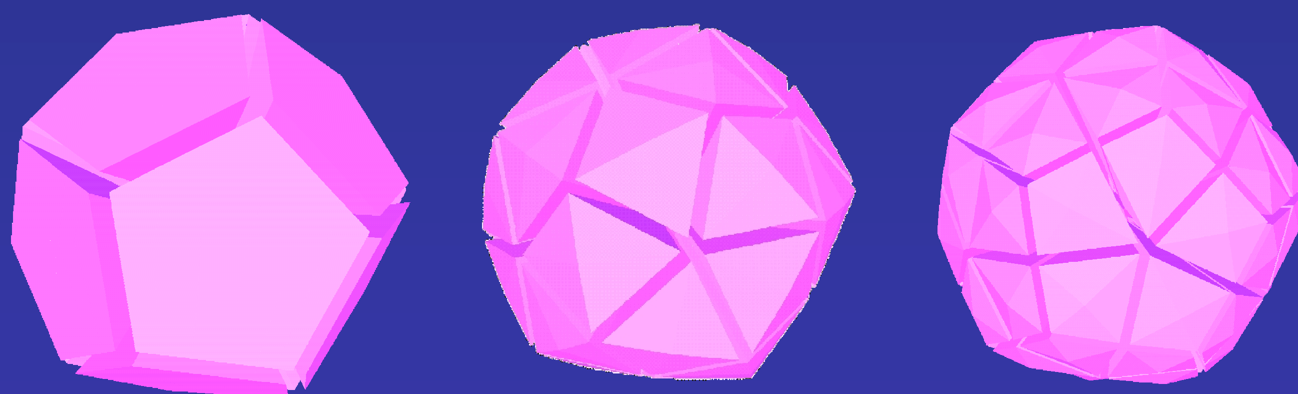


The Slow Growing Subdivision Applies to Very General Cells

- The coarse mesh must be a complex of topological balls



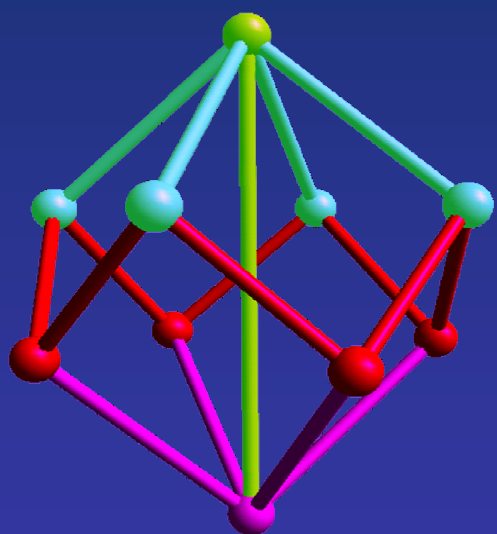
Tier 0 (base mesh)



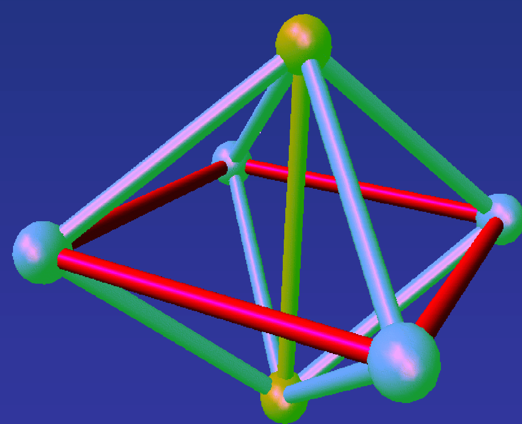
Tier 1 Tier 2 Tier 3

The Refined Mesh Is a Complex of "Diamonds"

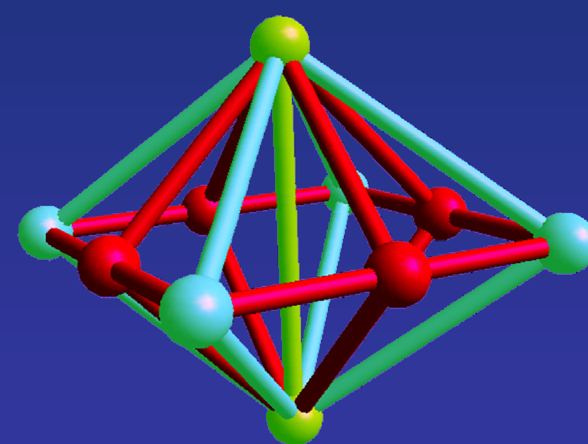
- **Diamond: topological ball that can be decomposed in a fan of simplices sharing a common edge.**



Tier 0

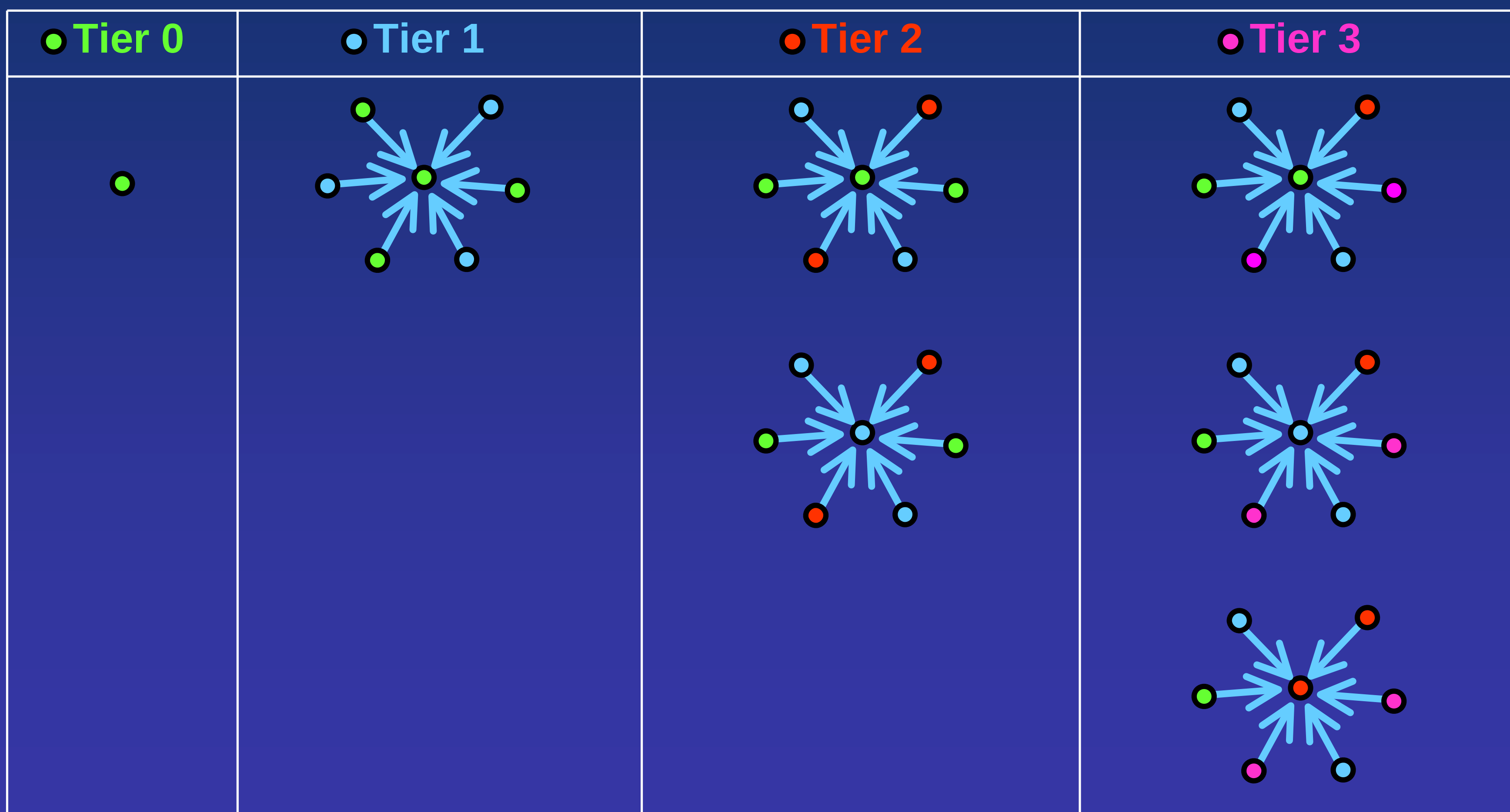


Tier 1



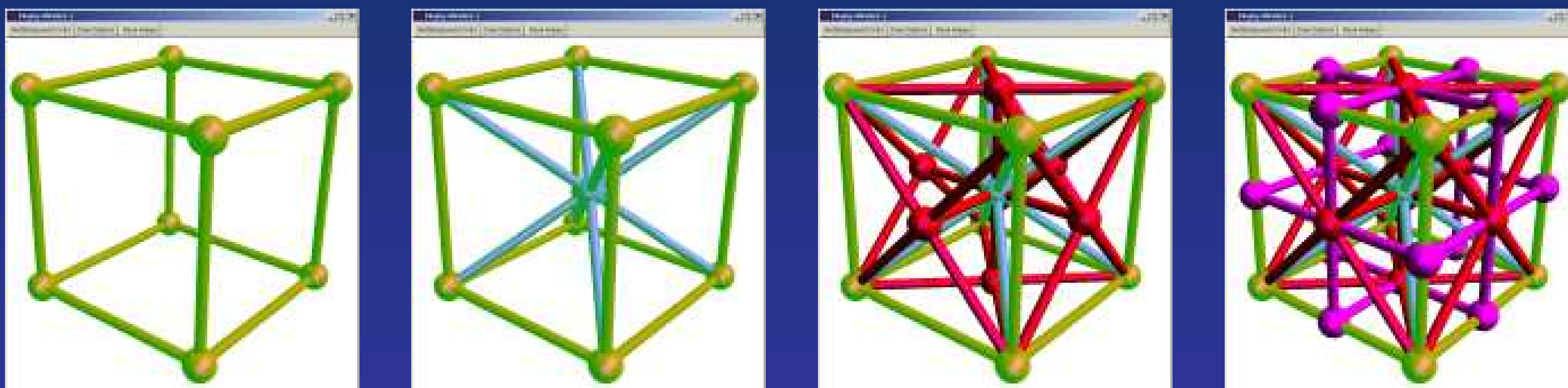
Tier 2

To Achieve Smooth Refinement We Need an Averaging Rule



Multi-resolution representation based on wavelets transform.

Lifting for $\sqrt[3]{2}$-subdivision meshes

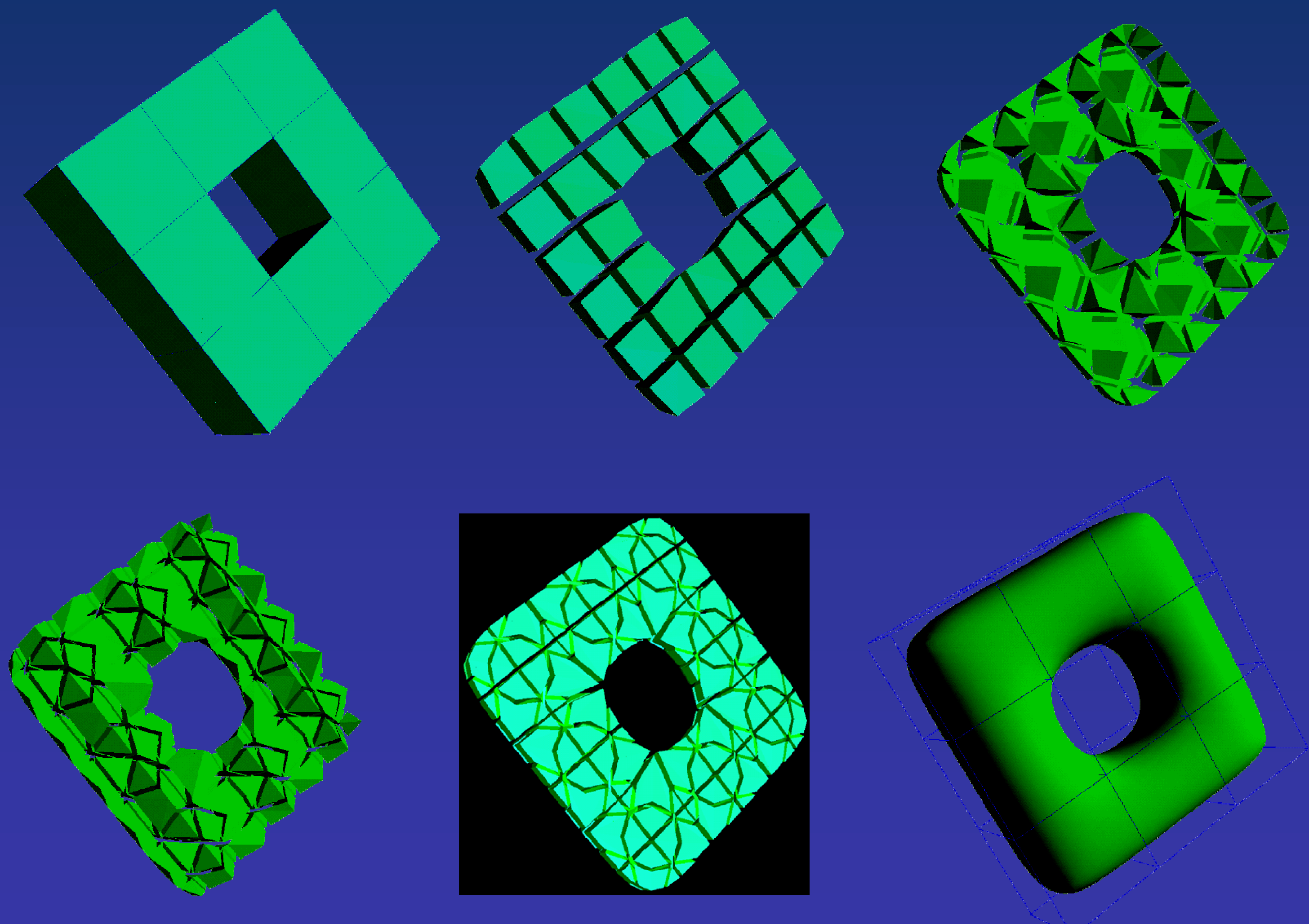


encoding

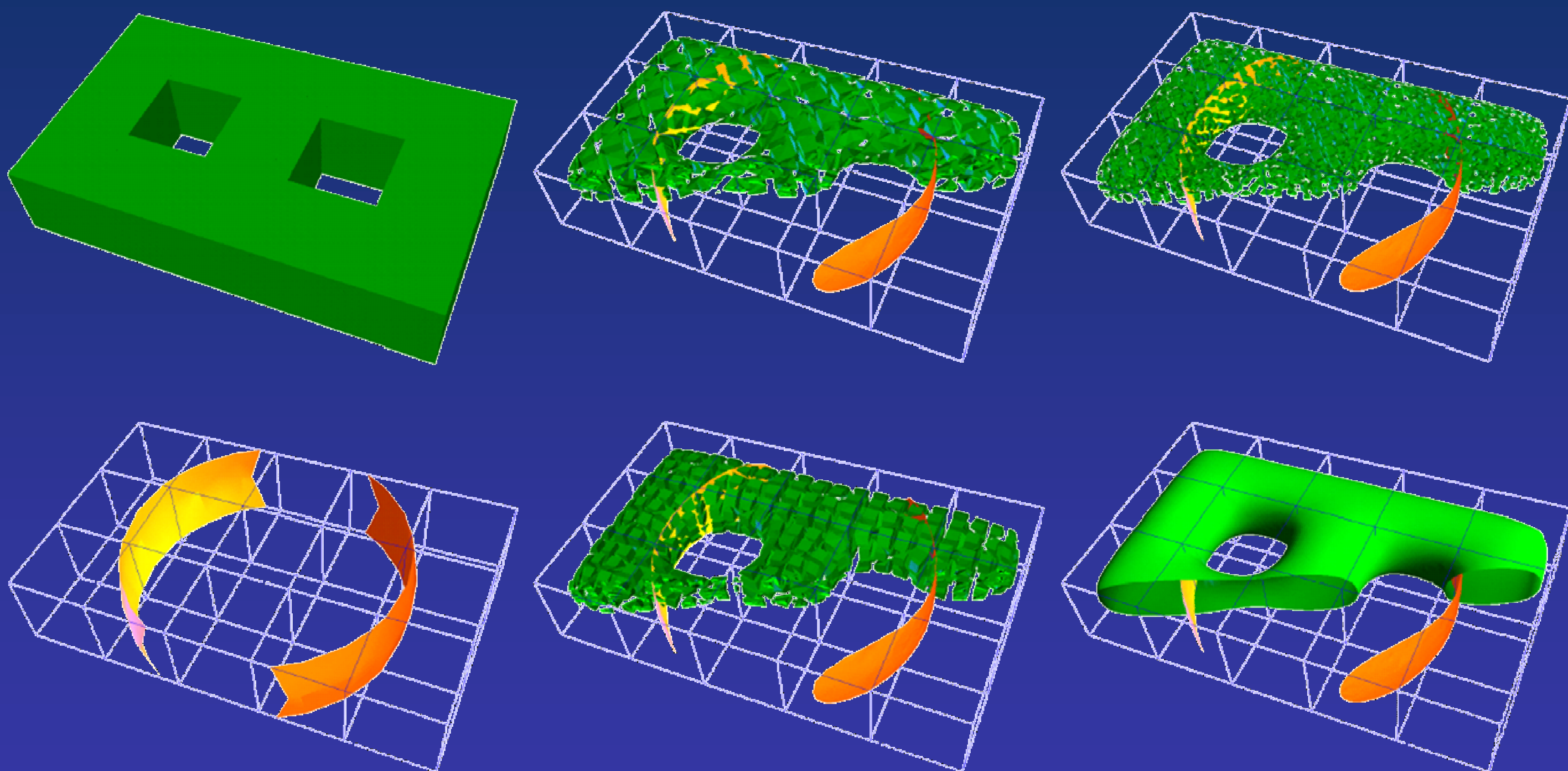
w-lift (α, β):	🔵 = α 🔵 + β 🟢	🔴 = α 🔴 + β 🟢	🟣 = α 🟣 + β 🟢
s-lift (α, β):	🟢 = α 🟢 + β 🔵	🟢 = α 🟢 + β 🔴	🟢 = α 🟢 + β 🟣

3D lifting: http://graphics.cs.ucdavis.edu/research/Multiresolution-Volume.html
4D lifting: http://graphics.cs.ucdavis.edu/research/Multiresolution-4D.html

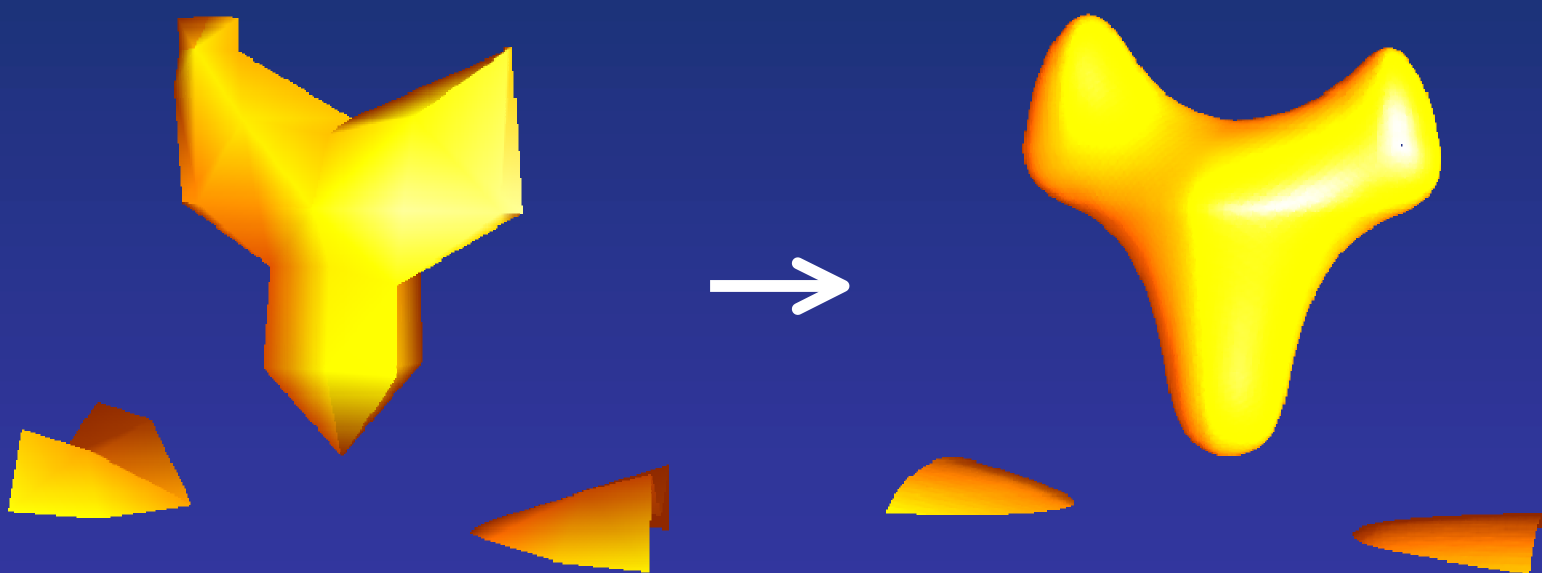
SGS Refinement of a Toroidal Mesh



Concurrent Mesh Subdivision and Field Smoothing

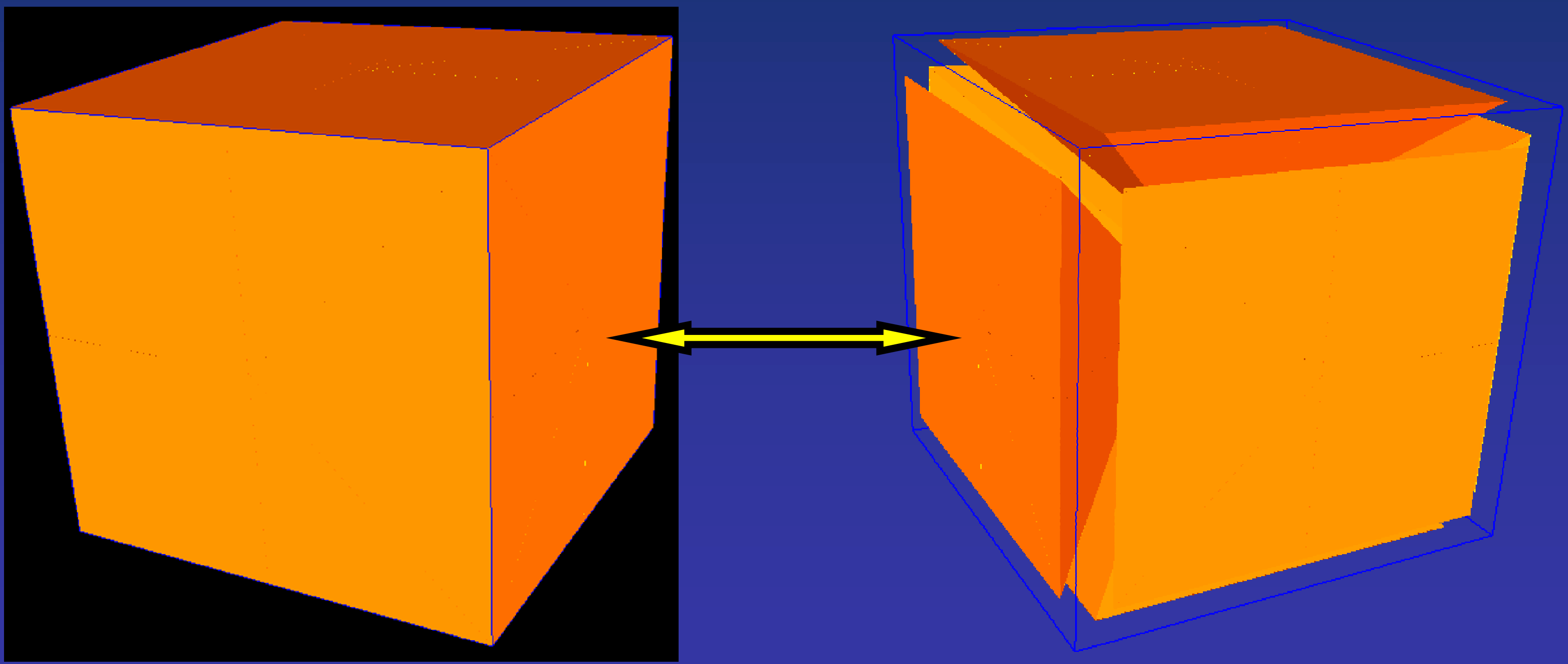


Indirect Isosurface Smoothing with Mesh Subdivision.

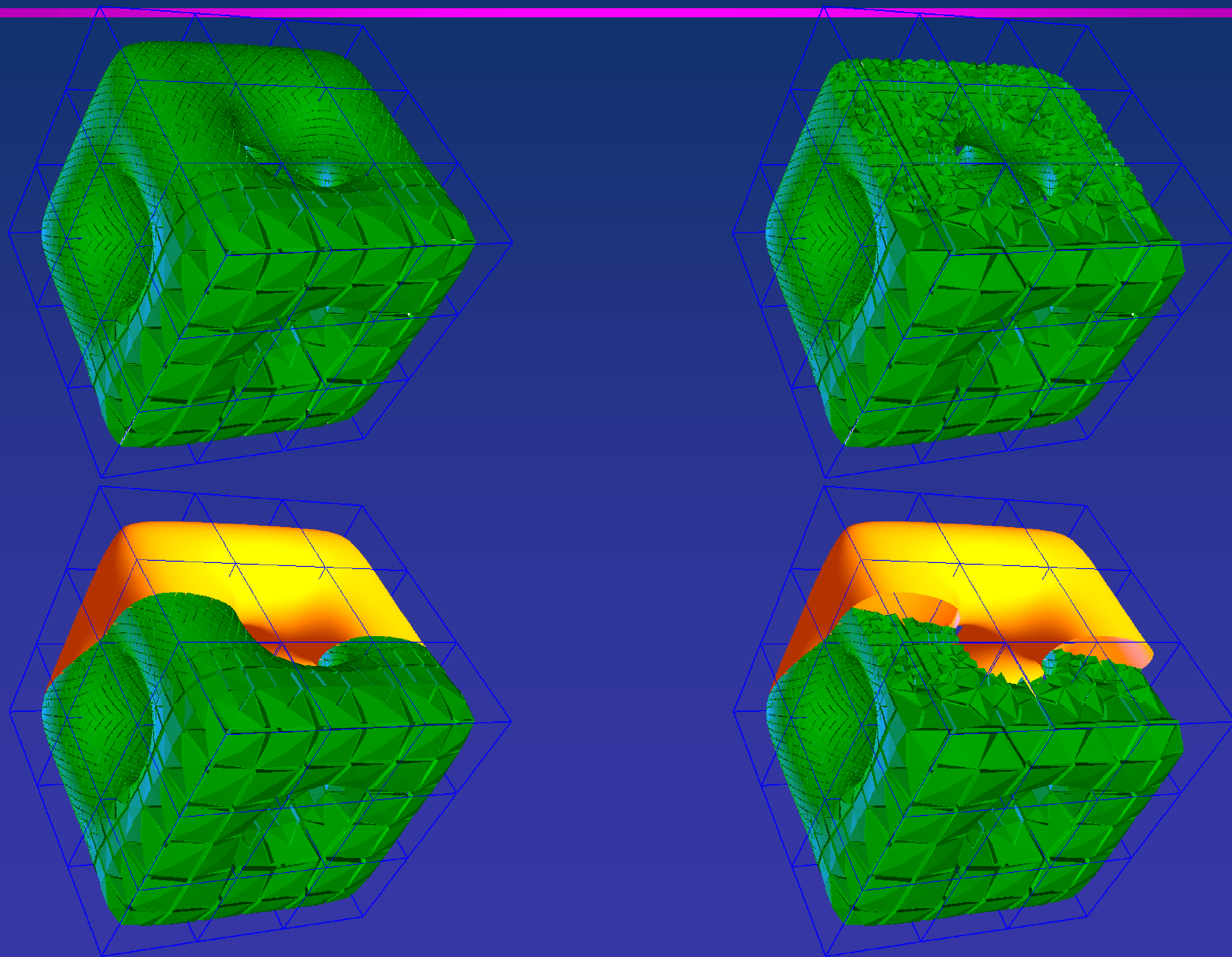


SGS Does Not Require Special Cells to Refine Adaptively

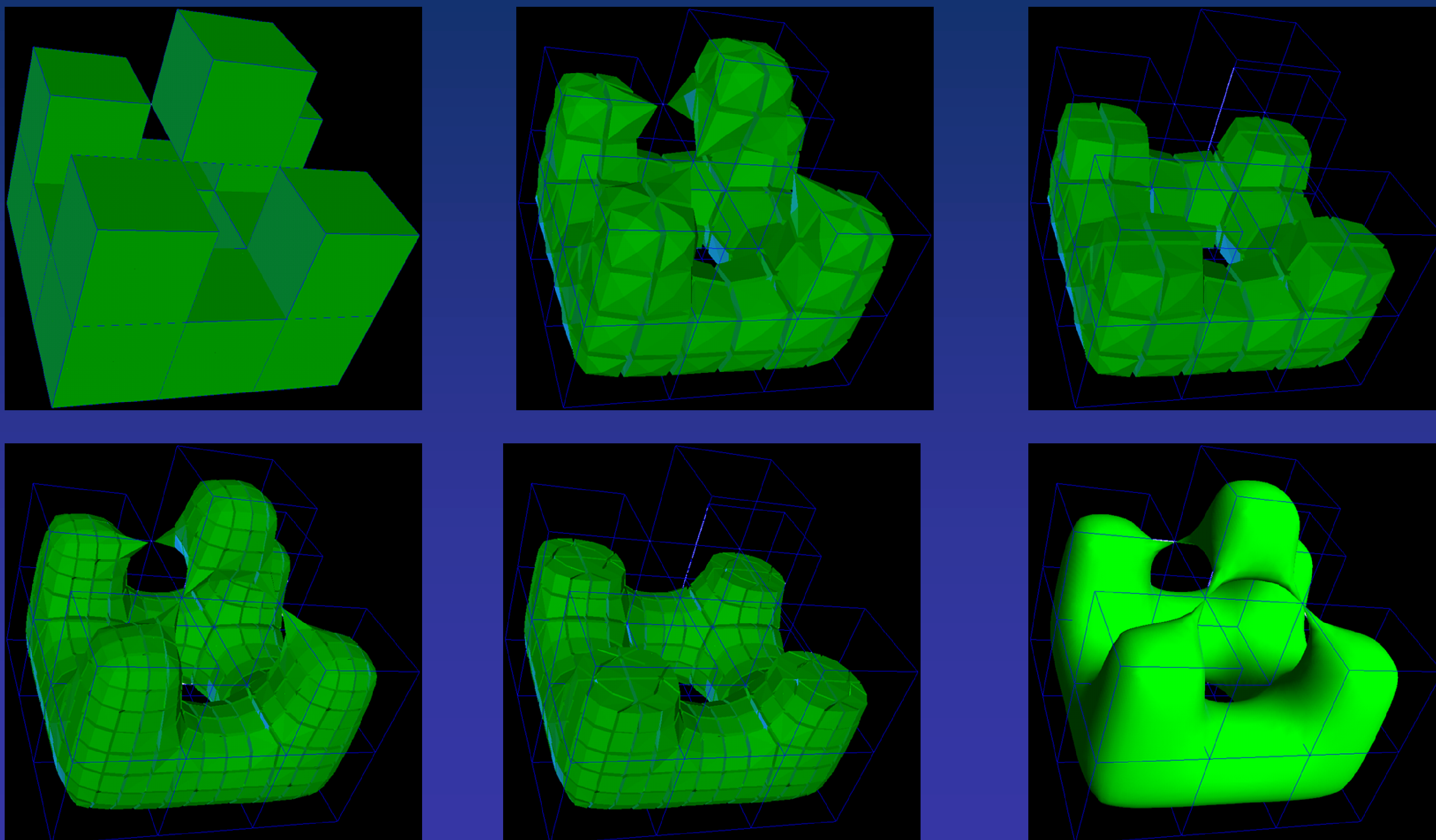
- Simply skip one "merge" step to connect cells at different level of resolution.



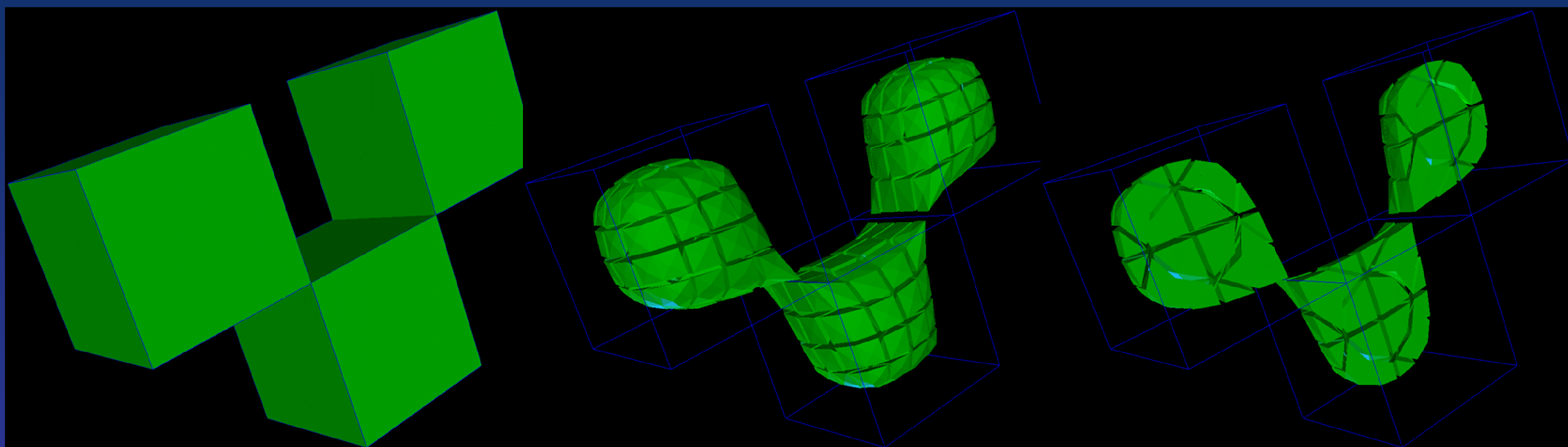
Adaptive SGS Refinement of a High Genus

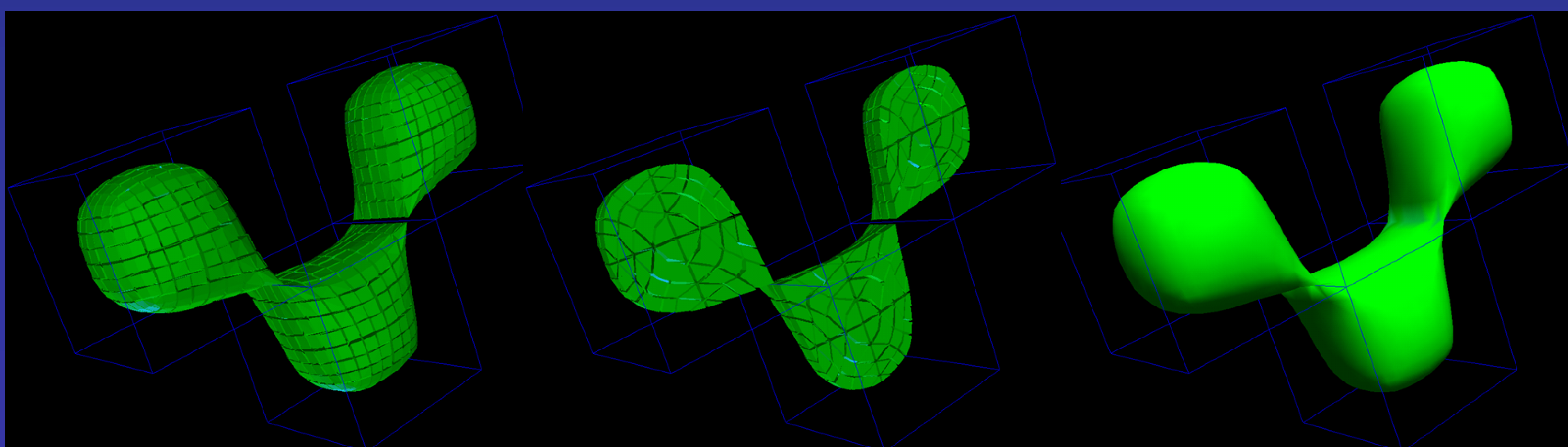


Non-manifold Vertices Do Not Need Special Treatment

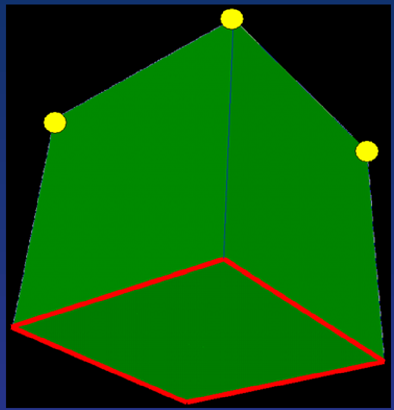


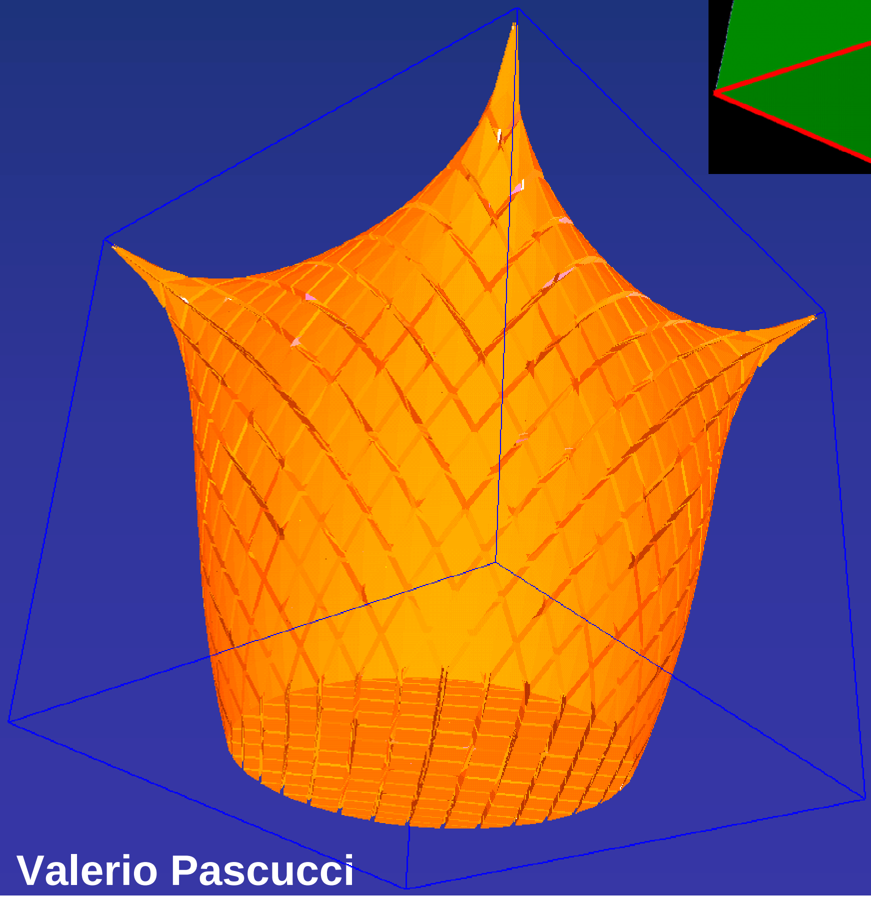
Non-manifold Edges Do Not Need Special Treatment

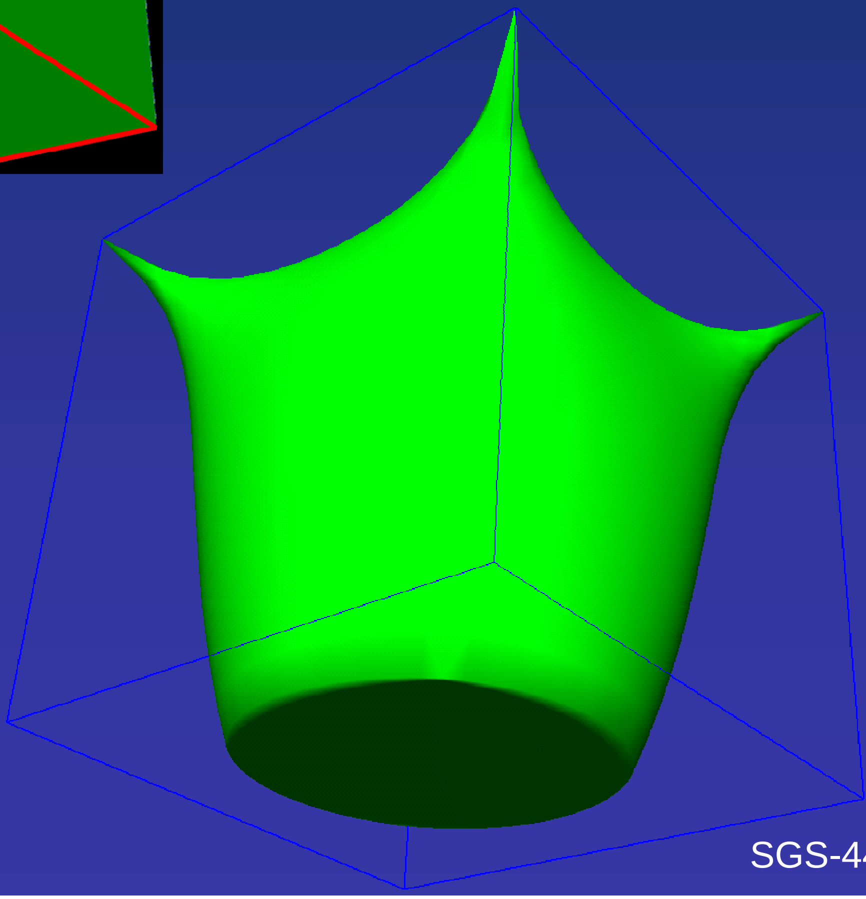




SGS of a Cube With Sharp Base Polygon and Four Sharp Vertices







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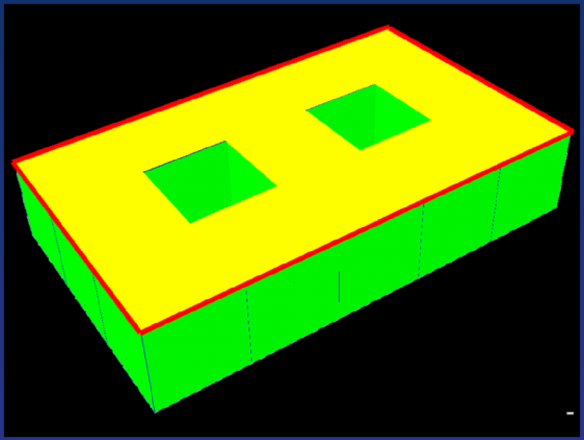
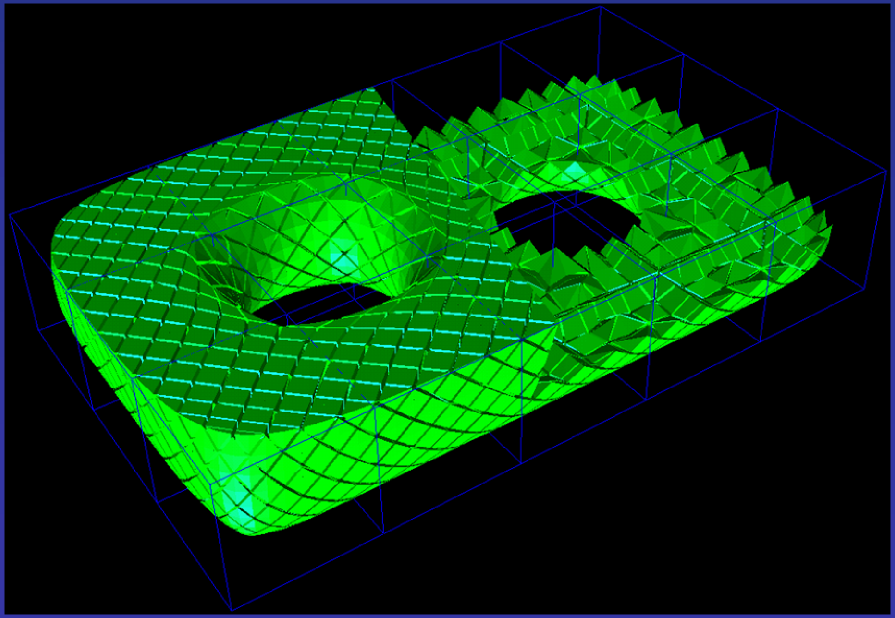
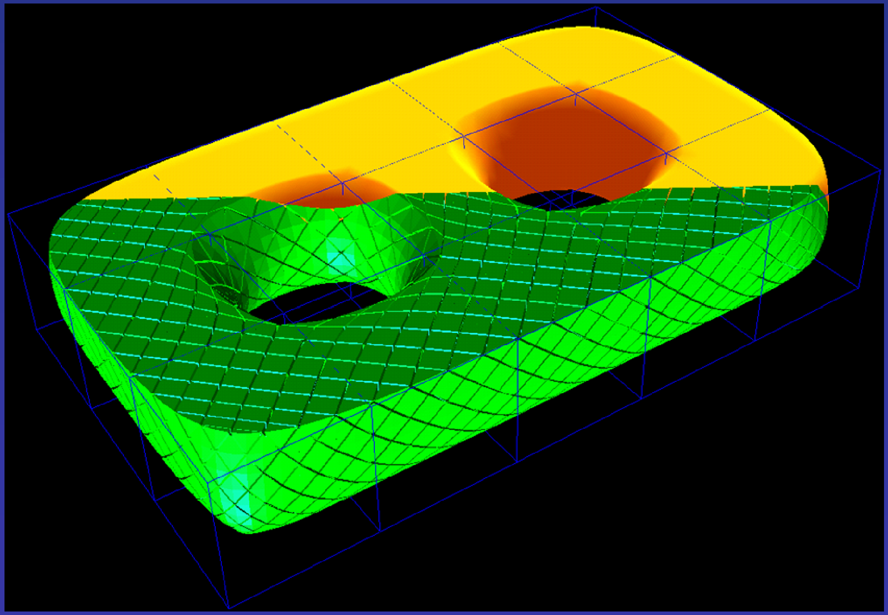
SGS Refinement of a Double Torus With a Sharp Polygon







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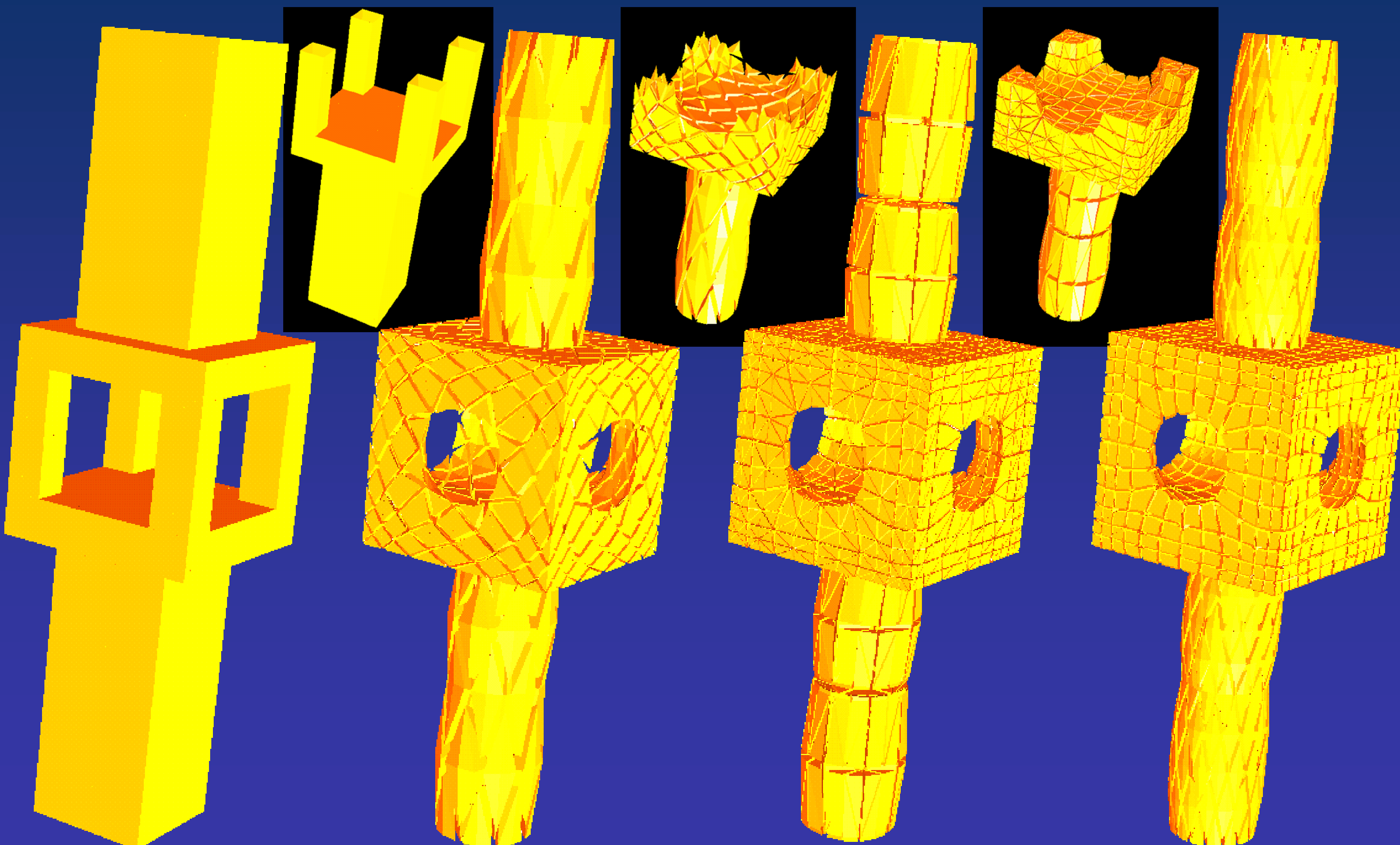
SGS Refinement of the Mesh of a Mechanical Part



Future Directions

- Smoothness analysis of limit mesh
- Re-meshing unstructured data
- Efficient multi-resolution representation
- Progressive compression

UCRL-PRES-200744

- This work was performed under the auspices of the U.S. Department of Energy by University of California Lawrence Livermore National Laboratory under contract No. W-7405-ENG-48.